Recommendation On Implementation Of Massachusetts Institute Of Technology

Research In Secondary Item Supply Control

Prepared By

Ordnance Inventory Management Project
Office Of Ordnance Research
March 1959

ABSTRACT

Title: RECOMMENDATION ON IMPLEMENTATION OF MASSACHUSETTS INSTITUTE OF TECHNOLOGY, March 1959

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This report covers an extension of the work previously reported under Ordnance Logistics Studies I and II, Massachusetts Institute of Technology, Spare Parts Supply Control, July 1957 and August 1958 (ASTIA Nos. AD-137 729 and AD-205 827). The inventory model previously described is revised in that the demand Y_t in a period of time t is treated as a Compound-Poisson random variable. In addition, the assumption made in ITR No. 9 that the difference between maximum stock (R + Q) and stock on hand (N) is a normally distributed random variable is relaxed and a revised solution for optimal values of R and Q is derived. A method for forecasting the variance-to-mean ratio, used in estimating the standard deviation of demand during the lead time, is presented which is based on a relationship noted during the course of the study between the variance-to-mean ratio of an item and the joint distribution of its mean demand and unit price.

This report also includes results of additional simulations performed after publication of ITR No. 9, estimates of the impact of implementation of the model in the Ordnance repair parts inventory system and a description of the computational procedure recommended for use.

Recommendation On Implementation
Of Massachusetts Institute of Technology Research*
In Secondary Item Supply Control

Prepared By
Ordnance Inventory Management Project
Office of Ordnance Research

*Massachusetts Institute of Technology Research
Performed Under Contract DA-19-020-ORD-2684
Department of Army Project No. 599-01-004
Ordnance Rand D Project No. TB-001
OOR Project No. 968 (Rev)

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SECTION I

RECOMMENDATIONS

The recommendation of the Ordnance Inventory Management Project that the Massachusetts Institute of Technology procedures for secondary item supply control be adopted is divided into two parts. These parts may be considered independently, as follows:

1. Safety Levels

It is recommended that the safety level component of the reorder point be calculated by the procedure described in Section II, below, for all Ordnance secondary items.

2. Procurement Cycles

It is recommended that, for the calculation of procurement quantities (cycles), the procedure described in Section II, below, be authorized as a basis for selecting economic procurement cycles so as to achieve least cost operation of the system.

These recommendations are discussed in Section II. In this discussion, the fundamental problems of supply management and their effects on supply performance, operating costs and inventory investment are examined.

Section III contains a detailed analysis of the impact on supply performance, operating costs, and inventory investment that can be expected from implementation of the M.I.T. procedures.

In Section IV will be found a critical evaluation of the M.I.T. procedures. Here are discussed some limitations of the proposed system, how they are treated under present procedures and suggested bases for further improvements.

Section V contains a more detailed description of the M.I.T. method and outlines the operating procedures by which the proposed system can be implemented. The appendices contain a discussion of how cost and performance estimates were calculated and a description of the mathematical model which formed the basis for these recommendations.

SECTION II

GENERAL DISCUSSION OF THE PROPOSED SYSTEM

Background

Work on this project began about three years ago when it was felt that a scientific study of the mechanics of the supply system might yield clues as to how supply performance might be improved. The techniques of operations research were believed to offer the best method of approach and the services of the Operations Research Project of the Massachusetts Institute of Technology were therefore enlisted. When the M.I.T. research studies began to show promise, their efforts were joined with those of operating Ordnance personnel with the establishment, in August 1957, of the Ordnance Inventory Management Project. The objectives of this project were to evaluate research findings and to formulate a basis for implementation of such findings as appeared to offer promise of substantial improvement of the supply system.

Early in the course of this study, it became apparent that safety levels were a fundamental problem. Extensive investigation was made of actual demand data on a representative sample of Ordnance secondary items. The wide fluctuations exhibited by these data made it abundantly clear that existing policies of fixing standard safety levels for broad classes of items would not allow achievement of desired supply performance. Extensive mathematical analysis, supplemented by large-scale computer simulations, resulted in the new method of determining safety levels that is proposed herein.

While the mathematical analysis was in process, the increasing interest in reduction of system operating costs, as evidenced by the 5 x 5 study, was recognized. A mathematical treatment of procurement and holding costs was incorporated to allow quantitative determination of economic procurement cycles where procurement on this basis is considered desirable. Cost data were collected and analyzed and estimates calculated of effects of economic procurement cycles on operating costs and inventory investment.

The final phase of this study was an investigation of techniques of implementation. Nomographs for simple determination of safety levels and procurement quantities were developed, along with recommendations as to operating procedures that could be used under either manual or ADPS operation.

Discussion of Proposed System Concepts

It is necessary at this point to discuss the fundamental notions of the proposed system, how they are related to concepts underlying present regulations, and why implementation of the proposed system will result in changes in supply performance, operating costs and inventory investment. A more detailed discussion of these concepts and the mathematical work underlying their development appear in the technical reports of the M.I.T. Operations Research Project.[1]

1. Safety Levels

The safety level computation under the proposed method introduces a new concept, namely the variance of demand. This concept recognizes that there is a random short-term fluctuation in demand and that the magnitude of this fluctuation differs from item to item, being generally larger for high demand items than for low demand items. It is this fluctuation that contributes largely to the unpredictability of demand and which makes the treatment of safety levels under present regulations inadequate for the purpose of achieving a given level of supply performance.

Under the proposed procedure, the short-term fluctuation is treated as a random process and its magnitude for each item can therefore be mathematically determined.

Given:

(1) The level of supply performance desired (expressed in terms of national availability desired for the item), and

(2) The procurement cycle desired (which can be the optimal economic procurement cycle described below)

the safety level (and thus, the reorder point) required to achieve the stipulated level of performance is uniquely determined. In actual practice, the safety level is obtained by reference to simple nomographs; under ADPS operation, this determination can be made easily within the computer.

1. These appear in Interim Technical Report No. 7 (July 1957) and Interim Technical Report No. 9 (August 1958) submitted to the Office of Ordnance Research under Contract No. DA 19-020-ORD-3684, Department of the Army Project No. 599-01-004 Ordnance Research and Development Project No. TB-001, Office of Ordnance Research Project No. 968 (Rev).

2. Optimal Procurement Cycles

The cost of operating the system is made up of two sets of costs, which operate in opposing directions. One is the cost of procurement, which consists of the cost of processing the procurement action and the fixed manufacturing cost which must be amortized each time the item is bought. This cost decreases as procurement quantities increase. The other is the cost of holding the item, which is composed of such cost elements as storage and handling, maintenance in storage, deterioration, obsolescence and interest on investment. This cost increases as procurement quantities increase. When these cost elements are known, it is possible to determine that procurement quantity (which may also be expressed in terms of a procurement cycle) that will reduce the sum of the procurement and holding costs to a minimum. This quantity is unique for each item; least cost operation cannot be achieved by establishing fixed procurement cycles for broad classes of items as is done under present procedures.

This concept is not new; essentially, it represents an extension to all items of the philosophy of economic procurement quantities expressed by the current 5 x 5 policy. The innovation in the proposed procedure is one of method, rather than concept, in that the economic procurement quantity is explicitly determined for each item by considering all relevant cost factors in a quantitative manner. In actual practice, economic procurement quantities on low cost items can be obtained manually from simple nomographs, or can be easily determined within the computer under ADPS operation. On medium and high cost items, the fixed manufacturing cost is frequently the overpowering cost element and an estimate of this cost element would have to be obtained. In addition, it would probably be desirable to obtain more exact estimates of holding cost elements, particularly obsolescence and deterioration, on certain items where these factors are important. While extra effort would be involved in obtaining these estimates, the potential savings in operating costs are of considerable magnitude. Moreover, this additional work would be required on a relatively small number of items.

It should again be emphasized that, while achievement of a given level of supply performance is independent of cost considerations, the selection of the procurement cycle, whether by present methods or by optimal procurement quantity computations, fixes the safety level required to achieve the desired level of supply performance. It is in this respect that the proposed procedures depart significantly from current practice.

3. Investment in Inventory

One additional factor must be considered in connection with examination of system performance and of system operating costs. This is the investment in inventory that must be made in order to (1) achieve a given level of supply performance, and (2) to do so at least cost.

A given level of supply performance can be achieved with almost any inventory investment one likes. For example, if a part costing $1.00 each is issued at the rate of exactly one unit a day, one can choose to reorder one unit of the part each day, in which case the average investment in working inventory would be $.50. Or, one can choose to order a quantity of 365 units only once a year, in which case the average inventory investment would be $182.50. Indeed, any reordering policy can be followed, each requiring its own investment in inventory.

Naturally, this holds true only so long as the issue rate is known and constant. As soon as uncertainty as to issue rates is introduced, the question of inventory investment becomes more complex. In order to maintain a given level of supply performance, one must provide a cushion of stock as protection against the element of uncertainty in the period between placement of successive reorders. This is, of course, the familiar notion of safety levels.

The addition of the safety level to expected rate of issue naturally requires that the investment in inventory be increased. Under current regulations, this safety level is thirty days' stock for low dollar value items and sixty days' stock for all others. The inventory investment is thus increased by that amount over what would be required if no element of uncertainty as to issue rates existed.

Under the proposed procedure, however, the concept of a standard safety level for broad classes of items is rejected since all evidence clearly shows that a known level of protection against uncertainty of demand cannot be obtained by that method. Instead, the desired level of protection is selected in advance and the safety level required to achieve that protection is computed for each item. This means that many items will require more than a thirty (or sixty) days' safety level, others less. Thus, a shift in inventory investment among items will result and, in addition, the total investment in inventory will change. However, before one can evaluate completely the magnitude of the change in total inventory investment that would be brought about by adoption of the proposed procedures, it is necessary to consider the question of how procurement cycles are to be determined.

Once a procurement cycle quantity is specified, no matter by what method, the range of possible inventory investment policies ceases to exist. One must pay in inventory investment not only for the safety stock but also for the procurement cycle quantity specified for the item. In other words, the establishment of a policy governing the selection of procurement cycle quantities fixes what the inventory investment in the item will have to be.

There are a number of alternatives open with respect to establishment of procurement cycle policies. However, only two will be discussed herein, namely, the policy specified by current regulations and the optimal procurement cycle of the M.I.T. policy procedures. Adoption of the M.I.T. safety level procedure can be accomplished under either procurement cycle policy but the effect on inventory investment will be different depending upon which procurement cycle policy is chosen.

Obviously, however, effective policies cannot be established without consideration of all the elements discussed above. In the following pages, therefore, the interactions of supply performance, operating costs and inventory investment are depicted in order to illustrate the policy courses that may be followed in implementing the M.I.T. procedures and the impact that the adoption of any policy can be expected to have on the supply system.

The M.I.T. Procedure

As indicated above, the recommendation is in two parts which can be considered independently. These are:

1. Computation of Safety Levels

The following must be known or specified in order to compute safety levels under the M.I.T. procedure:

a. National Availability Desired

This factor is the control variable and must be specified in advance by appropriate authority. It is expressed in terms of percent of time it is desired to be in stock nationally. For example, selection of a 99% national availability level would mean that it is desired that the item be out of stock nationally only 1% of the time.

b. Variance-to-Mean Ratio

This is a new factor. It is a measure of the unpredictability of supply demand in terms of frequency and size of orders. It can be looked upon as a reflection of customer ordering habits. The numerical value of this factor is determined from a nomograph which is entered with values of unit price and forecasted average demand for the item.

c. Procurement Cycle Quantity

This factor may be determined in any manner. It may be specified in the same way as under present regulations or in accordance with any other rules that may be prescribed. However, if least-cost operation of the system is desired, it must be calculated as indicated immediately below.

d. Unit Price

e. Procurement Lead Time

f. Forecast of Average Annual Demand

These are the same factors as are used in the present supply control study and are determined in much the same manner.

2. Computation of an Optimal Procurement Cycle

If least-cost operation of the system is desired, all the above factors must be specified or known and, in addition, the following must be estimated:

a. Procurement Cost

This is the sum of the administrative cost of procuring an item and the fixed cost of setting up for manufacture of the item.

b. Holding Cost

This is the cost of holding the item in stock and is composed of such elements as storage costs, deterioration charges, obsolescence and interest on investment.

Methods for obtaining estimates of these factors and details of the computational method, including numerical examples, are given in Section V.

National Availability and Its Effects On Operating Costs and Inventory Investment

The chart on the opposite page affords an overall picture of the various strategies that can be followed by those who formulate supply policies and the effects of policy decisions on national availability of stock, operating costs and inventory investments. The upper and lower curves depict the inventory investments and operating costs, respectively, that would result from least-cost implementation of the M.I.T. procedures at various levels of national availability. The two intermediate curves show the effects of implementation of only the safety level feature of the M.I.T. procedures with procurement cycles as specified by current regulations. Included for reference purposes are points indicating the theoretically calculated levels of inventory investments, operating costs and national availability that result from following present supply system policies at three points in time, namely (1) prior to implementation of the 5 x 5 policy, (2) after implementation of the 5 x 5 policy, and (3) after implementation of the policy permitting sixty-day safety levels for medium and high dollar value items.

Reference to the chart indicates that adoption of the 5 x 5 policy results in improvement in availability of stock, in that items are out of stock only 4% of the time rather than 8% of the time. Operating costs (costs of procurement and holding) decrease by two million dollars a year, but, on the other hand, investment in inventory increases by 24 million dollars.

Adoption of the sixty-day safety level policy results in decrease in time out of stock from 4% to 3%. This increased protection, however, results in a 3 million dollar increase in operating costs and a 22 million dollar increase in inventory.

A number of policy choices are open if the M.I.T. procedures are adopted. If, for example, the least-cost mode of operation is chosen and it is desired to set the national availability control at the 1% out-of-stock level, this could be achieved with a 12 million dollar increase in inventory investment. On the other hand, total operating costs would be decreased by 7 million dollars. Another possible least-cost policy would be to leave the present inventory investment unchanged. This would amount to fixing the national availability control at about 2.5% out of stock, as opposed to the current 3%. Operating costs would decrease, however, by 9 million dollars a year. Still another least-cost policy would be to retain the present 3% out-of-stock level of control. This would result in an 8 million dollar decrease in inventory investment and an 11 million dollar decrease in operating costs.

Similar observations can be made on the results obtained from implementing only the safety level feature of the M.I.T. procedure with procurement cycles determined by following current procedures. At the 1% out-of-stock level, operating costs would be just about the same as under present procedures and there would be only about a 3 million dollar increase in inventory investment. The benefits would be in decreasing average time out of stock from 3% to 1%.

A few words of caution are necessary in the use of this chart. First, the estimates of national availability, operating costs and inventory investment under the present procedures do not depict today's actual performance. They depict, rather, what would happen if current procedures were exactly followed. Actual performance data are affected by such factors as errors in forecasting, failure to make supply control studies at the proper time, use of mobilization reserve stocks as peacetime operating stocks, and a host of others. In addition, actual operating costs are impossible to determine since they are not yet available from the accounting system. Nevertheless, the chart serves a very useful purpose in that it compares what would happen under two sets of rules. The shifts in direction of national availability, operating costs and inventory investment can be accurately deduced even though their absolute magnitude may not be accurately determinable.

The second note of caution has to do with the items excluded from consideration in the chart. All items with a demand of less than 100 units a year are excluded, as are all items having a unit price of $500 and higher. These have been excluded since it is very difficult to assume what an intelligent supply analyst would do in the management of such items under either the present or proposed systems. The question is somewhat trivial in the case of low cost items having an annual demand of less than 100 units. Here any reasonable decision on procurement quantity would have little effect on operating costs or inventory investment. The question is much more difficult, however, in the case of high unit price items. Take, for example, the case of a $1,000 item which is used at the rate of only 3 units a year. Or, take as another example, a $1,000 item that is issued at the rate of 1,000 units a year. In the first example, the quantity procured would probably be a matter of negotiation with suppliers since it is hardly likely that the optimal procurement quantity would result in a practicable procurement action. This would probably also be true in the second case except that the possibility of a contract calling for continuous production of the item might be explored. These practices are undoubtedly being followed under the current system and would obviously be continued under the proposed system. The significance of the exclusion of these items from the chart lies in the fact that it would not be proper to make estimates of operating costs and inventory investments when the nature of the supply analyst's decisions cannot be divined under either set of rules. In any event, however, the proposed procedures offer a decided advantage even on such items. By taking into account the random fluctuation of expected demand, the supply analyst can fix his safety level to assure the desired degree of supply performance. Moreover, he would have available to him the analytical aids to help him to arrive at a reasonable procurement decision by taking into account in a quantitative manner the relevant cost factors.

National Availability and Safety Levels

As indicated previously, one singular feature of the proposed system is that each item has its own safety level -- a level which is a function of the random fluctuation of demand that can be expected for the item. The chart on the opposite page depicts an estimate of how the safety levels can be expected to very with the annual demand classifications of items. Safety levels prescribed under current regulations are also shown for purposes of comparison.

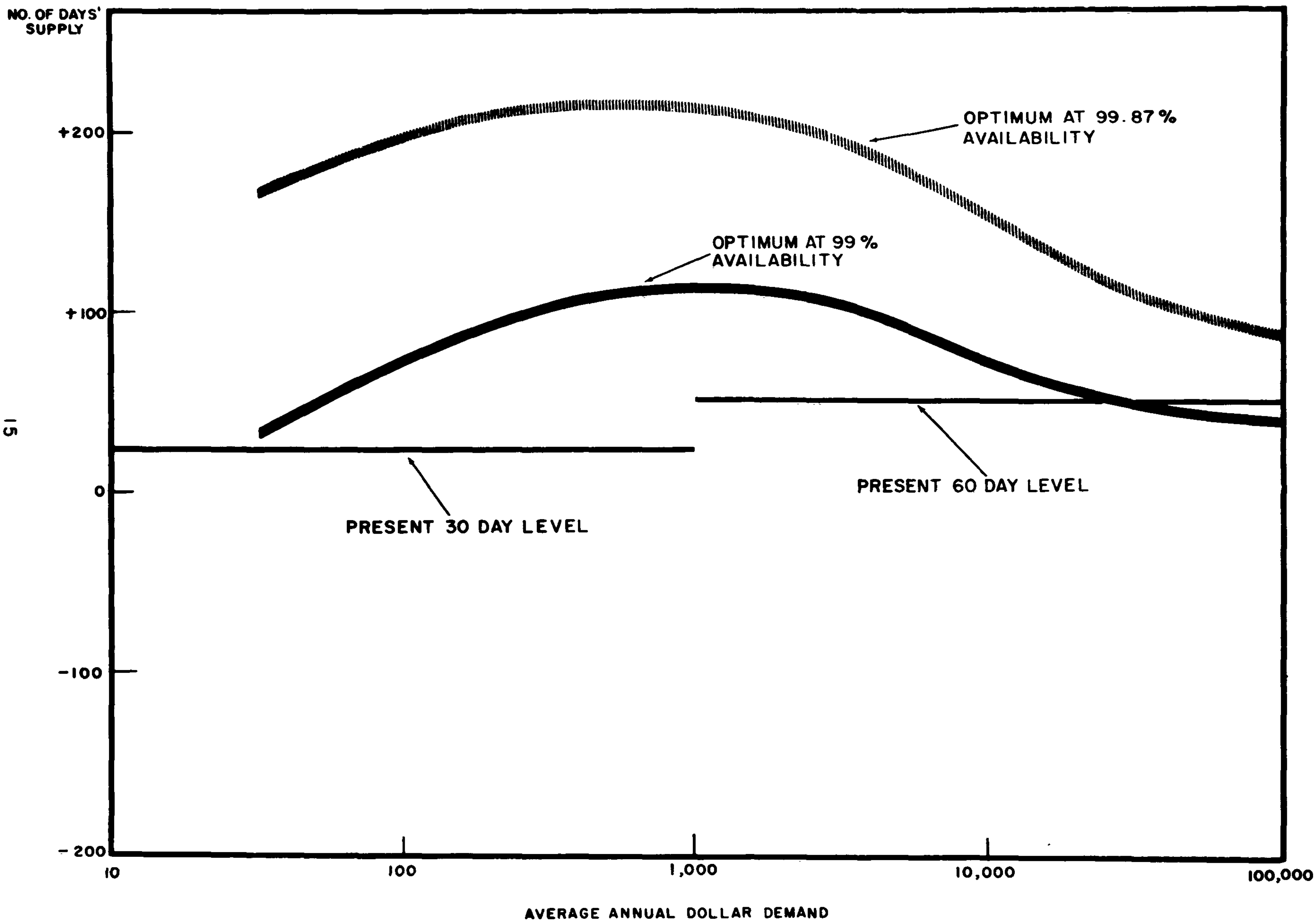
Under the proposed system, the level of national availability is set by competent authority. If, for example, the 99% level is chosen, this would be tantamount to setting a policy that accepts being out of stock nationally only 1% of the time as the performance norm. Examination of the chart indicates what would happen to safety levels under this policy. As can be seen, safety levels under the M.I.T. system would be higher than those under current regulations for items with an annual demand of less than $50,000. Safety levels under the proposed system at the 99% national availability level of control would be highest for items with an annual demand lying between $500 and $3,000. Beyond the annual demand rate of $3,000, the safety levels tend to become approximately equal to or slightly less than those presently prescribed.

The reasons for the shifts in safety levels that would result from adoption of the M.I.T. procedures are given in some detail in Section III. As will be seen in the detailed discussion, the safety levels are to a large extent a function of the customers' ordering habits. In analyzing the distribution of safety levels, it is much more informative to consider how they are related to the unit cost and quantities demanded on each item rather than the total annual demand for the item. The reason for this becomes clear when one considers the wide range of situations that are described by the statement, for example, that an item has an annual demand of $1,000. This could describe an item worth $1.00 which is demanded 1,000 times during the year, or it could describe an item worth $1,000 that is demanded only once a year. The range of order sizes is extremely important to the central question of how big the safety level should be to afford a given degree of protection. It can easily be seen that order sizes for high unit price items will generally be smaller and cover a smaller range than order sizes for low unit price items. Given the demand rates in terms of units demanded per year, it can reasonably be expected that the variance to mean ratio (which is a measure of the expected fluctuation in demand) will be smaller for high unit cost items than for low unit price items. Thus, safety levels for items in the former category will be smaller than those in the latter.

This finding has suggested that the relationships between the unit price, quantity demanded and the variance-to-mean ratio can be employed as a useful demand forecasting tool. A more detailed discussion of this concept of the variance-to-mean ratio will be found in Section IV.

EXHIBIT 2

AVERAGE OPTIMUM SAFETY LEVEL VS AVERAGE ANNUAL DOLLAR DEMAND



National Availability and Procurement Cycles

The chart on the opposite page depicts how average procurement cycles would vary under the M.I.T. procedures. The cycles indicated for the proposed system are based upon ordering the optimal procurement quantity after consideration of all relevant costs of procuring (including fixed manufacturing costs) and holding the item. Only the 99% national availability level of control is shown since procurement cycles in the range of 99% - 99.87% level of control are very nearly the same. Procurement cycles prescribed under current regulations are also shown on the chart for purposes of comparison.

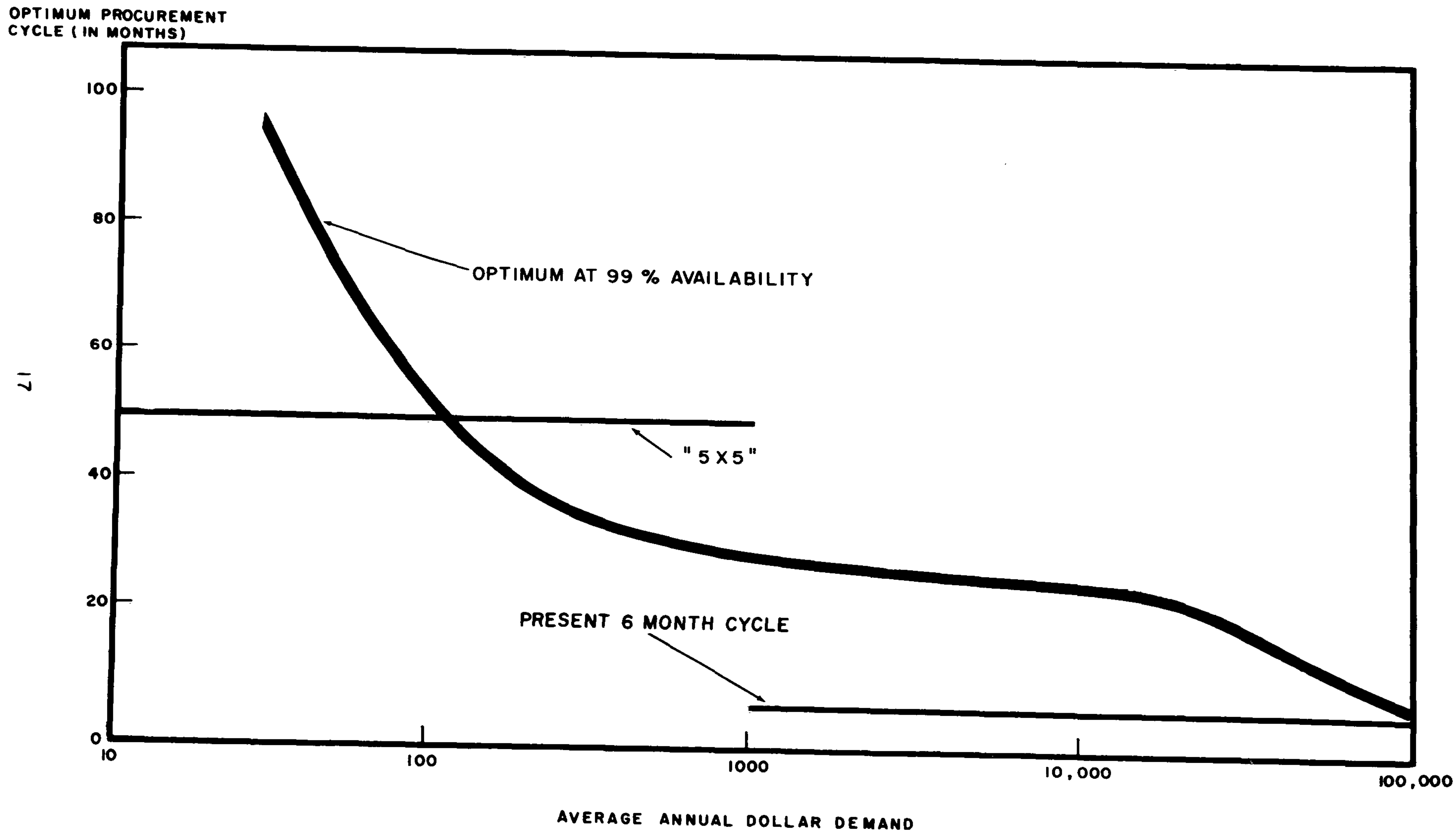
It can be seen that procurement cycles under the least-cost application of the M.I.T. procedures will differ significantly from those prescribed by current regulations. On items with an annual demand of less than $100, they are longer and thus larger quantities would be procured than at present. In the range of $100 - $1,000 annual demand, the quantities procured would be less. In the range of $1,000 - $100,000 annual demand, the quantities procured would be significantly greater due largely to the size of fixed manufacturing costs.

Here again, it is more meaningful to consider not the annual dollar value of demand for the item but its unit cost as well. It can logically be expected that high unit cost items will generally carry along with them a high manufacturing set-up cost. In such cases, a larger buy is indicated if least-cost operation is desired. This is merely the familiar notion of quantity discounts. Under the M.I.T. procedures, this factor is considered. In addition, however, the cost of holding is also considered and these two opposing cost factors are balanced to achieve a minimum operating cost.

No estimates of optimum procurement cycles are shown for items with an annual demand of greater than $100,000 or less than $20. The reasons for this exclusion are the same as given previously, namely that exigencies of the market place can be expected to affect the supply analyst's decisions to a considerable extent. It is considered that it would be improper to speculate on what his decisions would be under either the present or proposed procedures.

EXHIBIT 3

AVERAGE OPTIMUM PROCUREMENT CYCLE VS AVERAGE ANNUAL DOLLAR DEMAND



Benefits To Be Achieved Under the M.I.T. Procedures

The benefits that can be expected if the M.I.T. procedures are followed can be summarized as follows:

a. Protection against going out of stock nationally can be fixed at a desired level as a matter of policy with every reasonable assurance that the level of performance thus prescribed will be attained and maintained.

b. Tighter control will be maintained over national availability on a by-item basis. While national availability under present regulations is shown as being an average of 97%, many individual items are considerably below that average and many are above. Under the M.I.T. procedure, the great majority of items will cluster about the national availability average.

c. Policy decisions on how to apportion the inventory investment among items can be made on a logical basis, and the effects of such decisions on system performance and operating costs can be estimated in advance.

d. Performance in terms of initial fill of requisitions, on-time shipments, and customer waiting time can be improved by establishment of appropriate levels of national availability. In addition, the costs of improvement in terms of operating costs and inventory investment can be estimated in advance.

e. The number of back orders, shortage reports, extracts and emergency requisitions can be reduced by raising the national availability level of control to appropriate levels.

f. Assurance can be had that mobilization reserve stocks will be available for their intended purpose and that they will be used as peacetime operating stocks only to the degree specified by policy decision.

g. Customer ordering habits can be expected to improve as supply performance is improved, thus pointing the way towards operating the supply system at lower cost and with less inventory.

h. Significant savings in operating costs can be achieved if the least-cost mode of operation is adopted.

i. Supply analysts will have available to them a set of analytical tools to help in their decision-making function. As they acquire experience in their use, alternate courses of action can be considered and their effects estimated prior to formulation of decisions.

j. Supervisory review of supply analysts' judgement-type decisions will be possible on a more factual and reasonable basis.

SECTION III

DETAILED COST AND PERFORMANCE COMPARISONS

For the purpose of exhibiting the effects of demand variability on performance, the effect of the elements of operating costs on total costs, and further, of synthesizing the results to obtain a comparison of operations under the present and the proposed systems, an extensive analytical study has been completed beyond that described in Interim Technical Reports Nos. 7 and 9 of the M. I. T. Operations Research Project (see Appendices). Those reports described the method by which the mathematical basis for the proposed system was derived and the tentative conclusions reached on performance and costs comparisons after considerable analytical study and computer simulation of the present and proposed systems. The comparisons presented below include considerations not treated in previous reports, such as the effects of the application of the "5 x 5" policy, of the 60 day safety level for medium and high dollar value items, of manufacturing set-up costs and of the joint relationships of demand, demand variance, and unit price of the items. These newer results, therefore, are believed to constitute a more rigorous and meaningful basis for comparison of system performance than it has been possible to achieve previously.

A word of caution is necessary before the comparison results are discussed. It must be remembered that these results have been obtained through the operation, by analytical and computer simulation techniques, of two <u>supply systems</u>. It is not possible to compare what is <u>actually</u> happening today with what would happen if the proposed system were implemented because:

a. The present system has been changing frequently and many of the rules presently in effect have not been operating long enough to permit determination of their effects, and

b. Changes in reporting requirements and definitions prevent establishment of a stable set of base performance data for the present system, and

c. There is abundant evidence that present rules are not being rigorously followed in the actual management of the system.

Performance

1. Measure of Performance

While national availability is a nominal measure of performance, for many reasons it turns out to be better to think of national availability as a control, rather than a measure of performance. Clearly performance should be measured in terms of the promptness with which real customer need is met by the supply function. This, after all, is the mission of supply.

"Promptness" is not simple to measure. Three measures which partially but not completely describe promptness are:

(1) % initial source fill, approximately equal to depot availability.

(2) Average wait, in days, for fill when secondary source (extracting) must be used.

(3) Total "waiting", as measured by item-years of wait per year.

Exhibit 4 displays three possible patterns of filling customer demands with varying degrees of "promptness". It is by no means obvious which of these is the "best" pattern.

Nevertheless, national availability corresponds quite closely to percentage of on-time shipments, and leads to a simple, yet rigorous, solution of the inventory control problem. Simulations have proved its value as a control variable, as will be seen later in this section.

2. Factors Affecting Performance

The factors affecting national availability, the control variable for supply performance, are the following:

(1) Average demand during the procurement lead time

(2) Variance of demand during the procurement lead time

(3) Length of the procurement cycle

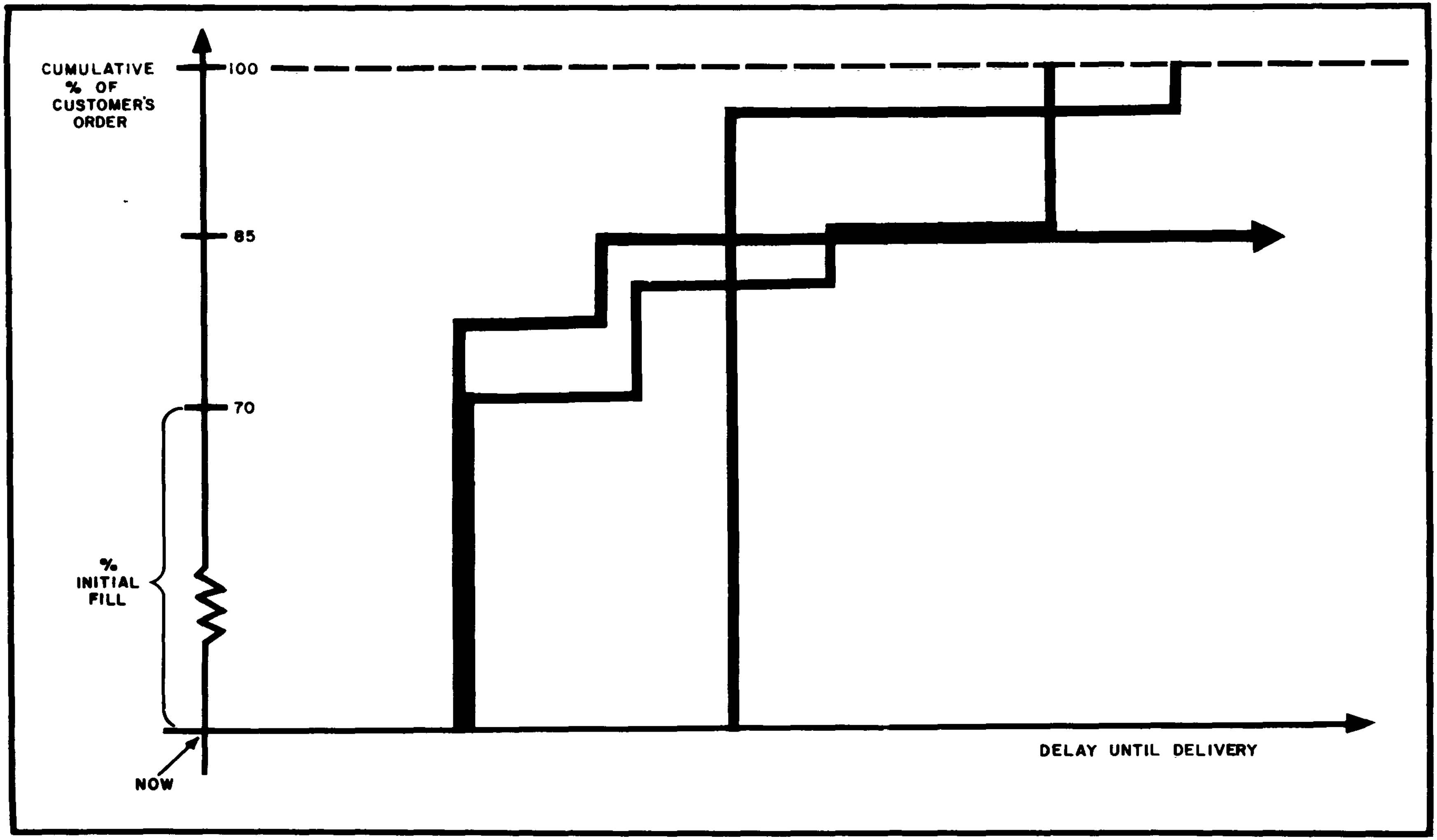
(4) Length of the procurement lead time

Policies regarding the allocation of national stock to the various depots, including extracting and stock leveling, obviously have a bearing on the promptness with which post, camp and station demands are satisfied, but have only indirect effects on national availability. These indirect effects are insignificant at high levels of availability (above 90%). It is assumed throughout that depot assets are accurately known.

It should be noted that only factor (2), the variance of demand during the lead time, is not presently used in supply control studies. Methods of estimating this factor are given in Section V of this report.

EXHIBIT 4

"PROMPTNESS"



3. Performance Comparisons

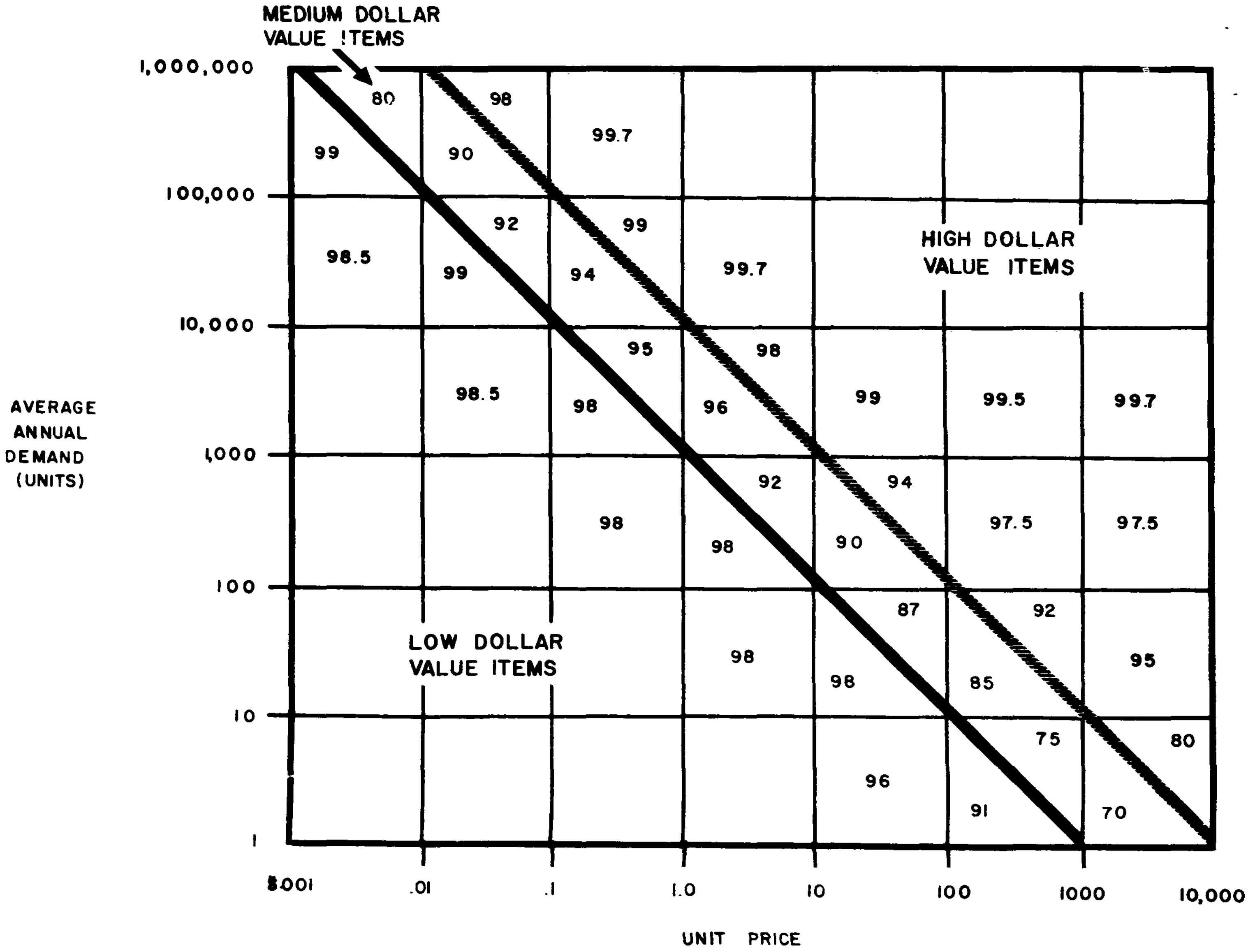
On the average, present procurement cycles and safety levels should yield the sort of national availability shown in Exhibit 5. For example, the average item with a demand rate of 500 units per year, and a unit price of $.50, procured under the 5 x 5 rules, will have an average national availability of 98%.

Certain general conclusions can be drawn from this chart. Medium dollar value items have considerably poorer availabilities than low and high dollar items. Also, most low dollar items have quite good availabilities. This, of course, is due to the long procurement cycles of the 5 x 5 policy.

Use of the proposed rules for determining safety levels would yield a uniform availability over the entire range of items at whatever level was specified.

EXHIBIT 5

REPRESENTATIVE PRESENT SYSTEM AVAILABILITIES VS. DEMAND AND PRICE

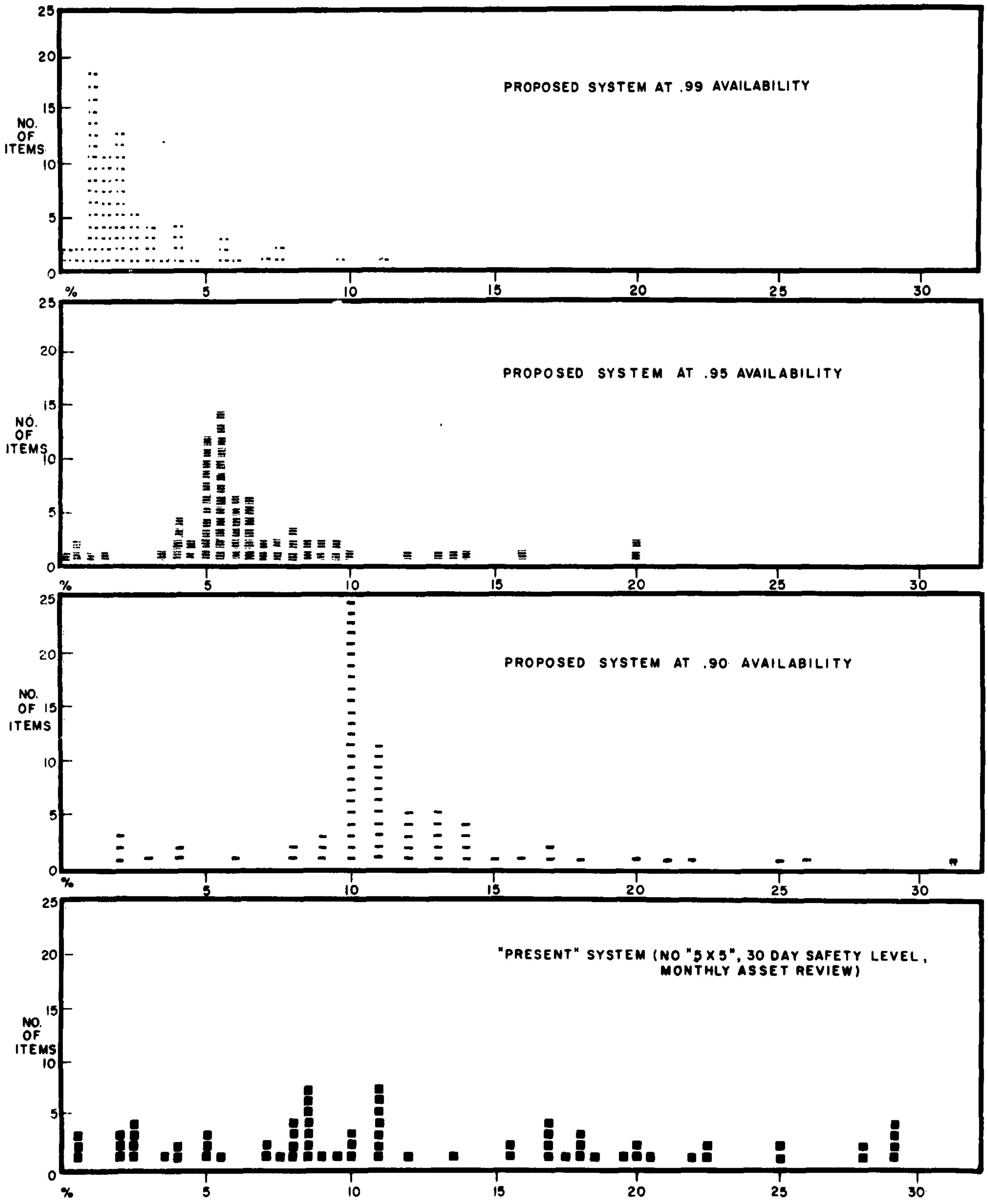


4. Reliability

In comparing the performance of two systems, one is usually interested not only in average performance but also in reliability of performance. Exhibit 6 shows the deviations from average performance generated in computer simulations of the proposed system and "present" regulations (before adoption of 5 x 5 and 60 day safety levels). Much the same results could be expected from a simulation of the present system, including 5 x 5 and 60 day safety levels. The average would be higher, but the dispersion would be quite similar to that shown.

EXHIBIT 6

DISTRIBUTION OF SIMULATED NATIONAL AVAILABILITY



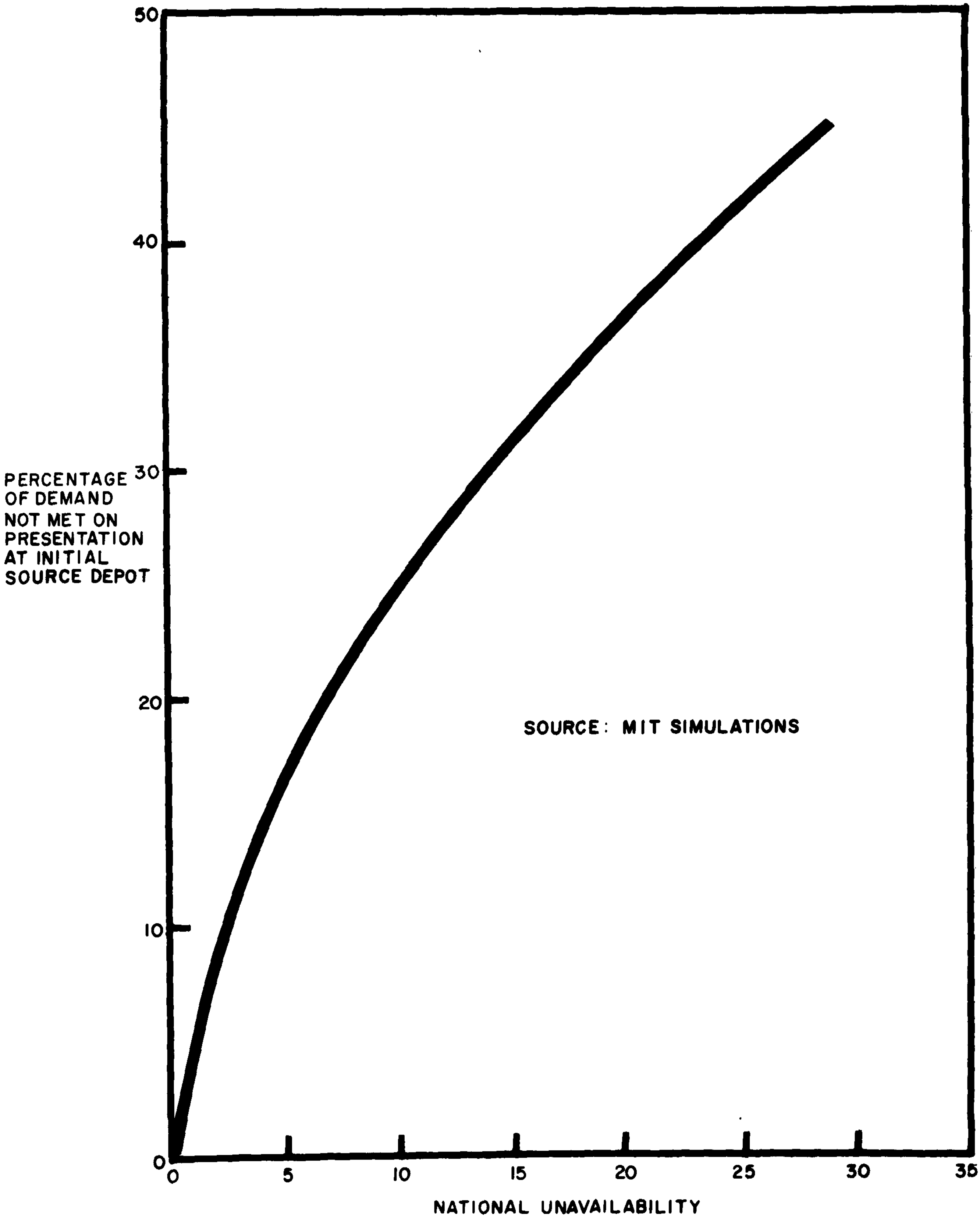
5. Average Depot Availability

Simulations of the present and proposed systems also generated data which show the relationship between depot availability (initial source fill) and national availability. Then results are contained in Exhibit 7, and are based upon present extracting and stock leveling rules.

Depot availability must, of course, be less than national availability. Note especially that at 99% national availability, depot initial fill will be only 92% on the average.

EXHIBIT 7

DEPOT OUT-OF-STOCK VS NATIONAL OUT-OF-STOCK



6. Distribution of Depot Availability

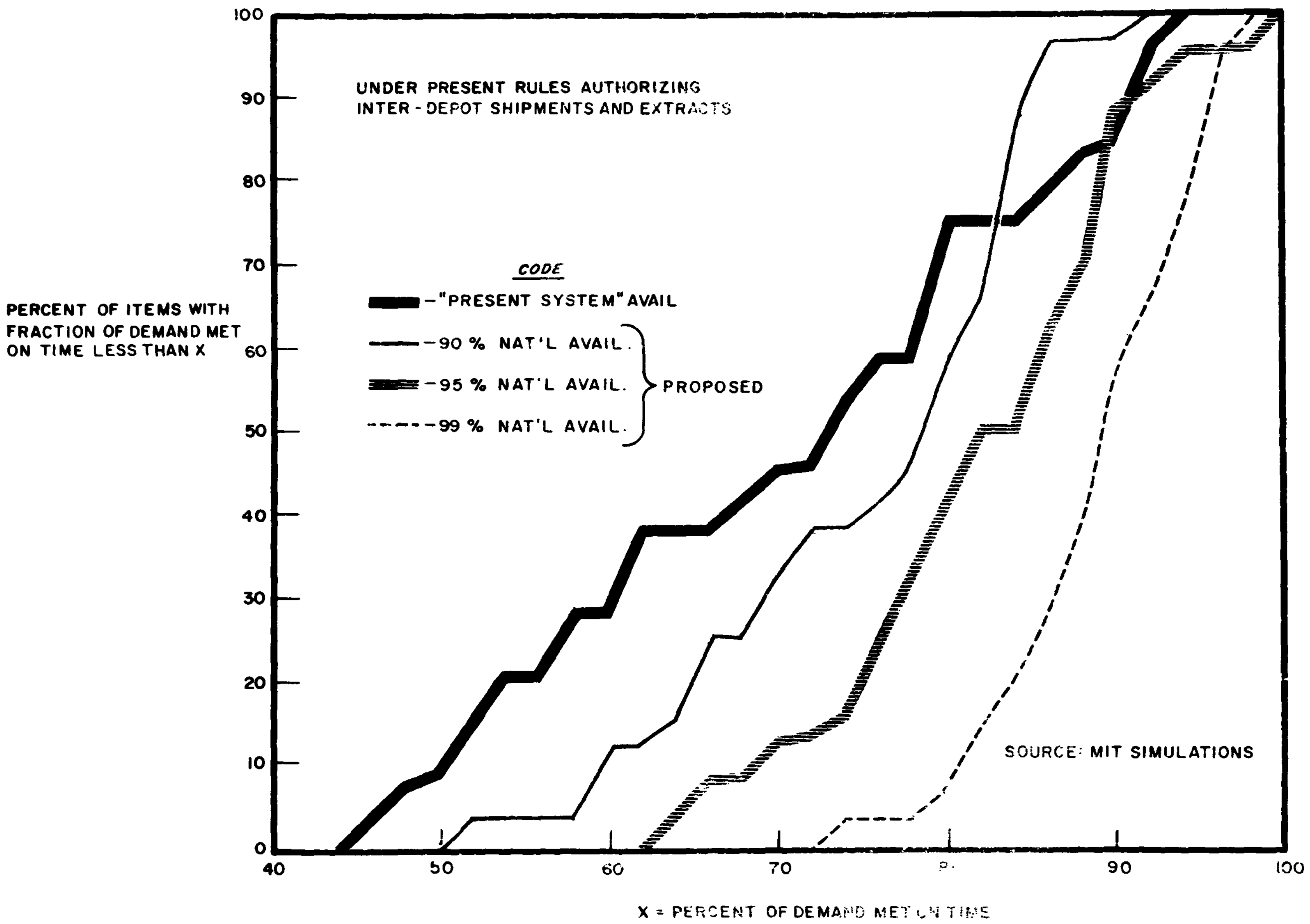
The way in which depot availability varies among items is shown in Exhibit 8. The previous exhibit displayed average depot availability. This one displays its range and dispersion.

The following chart is taken from this exhibit and shows median depot availabilities and minimum depot availabilities. (The median is that point above and below which 50% of the observations lie.)

	National Availability			
	90%	95%	99%	"Present"
Median Depot Availability	79%	83%	89%	73%
Minimum Depot Availability	50%	62%	72%	44%

EXHIBIT 8

PERCENT OF DEMAND MET ON PRESENTATION AT INITIAL SOURCE DEPOT



7. <u>Average Wait</u>

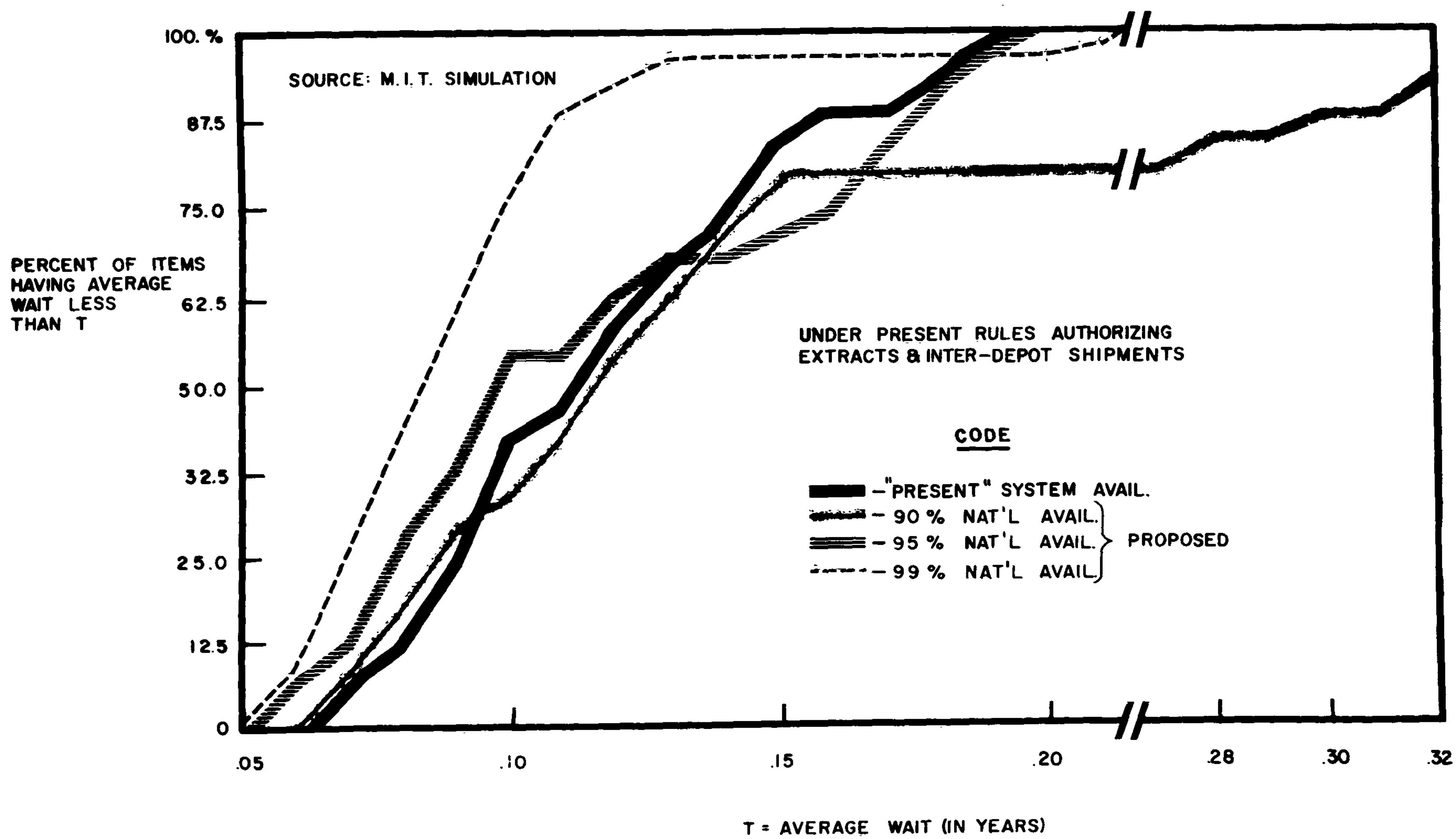
Another measure of "promptness" is the average wait for demand <u>not met on presentation</u> at the initial source depot. Simulation results are shown in Exhibit 9. Again, the "present" system is without "5 x 5" and 60 day safety levels.

Median average waits from this exhibit are as follows:

National Availability	Median Average Wait (Yrs.)
90%	.1175 (43 days)
95%	.0975 (36 days)
99%	.085 (31 days)
"Present"	.1140 (42 days)

EXHIBIT 9

AVERAGE WAIT FOR DEMAND NOT MET ON PRESENTATION AT INITIAL SOURCE DEPOT



8. Total Wait

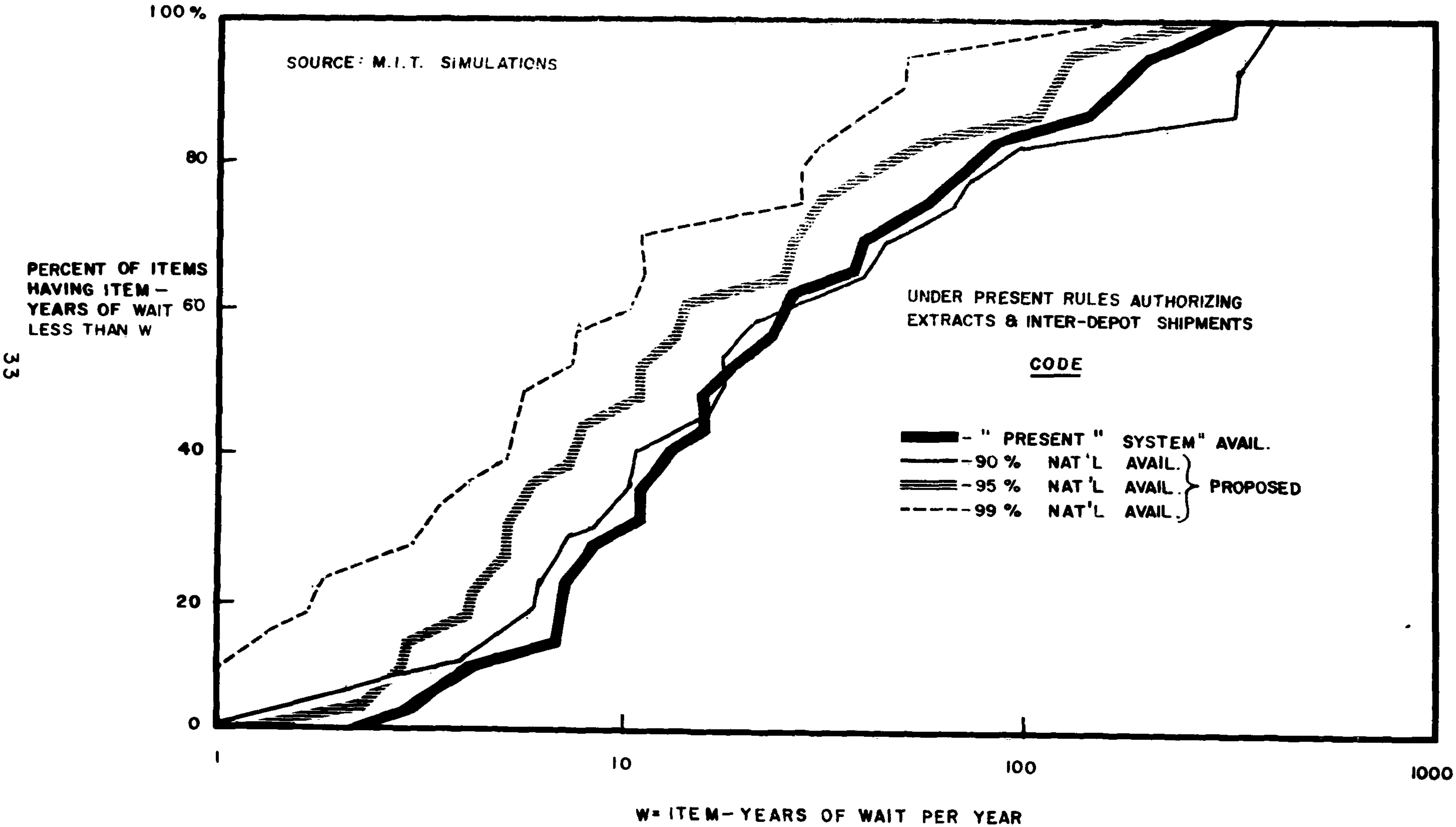
Exhibit 10 is concerned with total waiting rather than average wait. It shows the cumulative percentage of items having up to any given amount of item-years of wait per year. By this measure, a one month delay in shipping 12 units of an item is equivalent to a 6 month delay in shipping 2 units of the same item.

Median total waits from this exhibit are as follows:

National Availability	Median Total Wait (Item-Years per Year)
90%	17.0
95%	11.0
99%	5.6
"Present"	16.0

EXHIBIT 1[illegible]

ITEM – YEARS OF WAIT PER YEAR



9. <u>Conclusions on Performance</u>

National availability is an excellent control variable for most, if not all, reasonable measures of "promptness" of supply.

In addition, national availability should correspond closely to percentage of on-time shipments.

Even allowing for the improvements made by "5 x 5" on low dollar value items and 60 day safety levels on medium and high dollar value items, substantial improvements in performance are possible by going to the proposed system at availability levels of 99% and above.

These improvements lie not only in raising average performance levels, which could be accomplished by merely raising safety levels, but also in providing more consistent and reliable performance, and in making available to management the ability to specify performance in advance and to calculate its cost and inventory implications.

Operating Costs and Inventories

1. Measures of Costs

The two kinds of costs considered in this study are holding costs and procurement costs. Holding costs are the costs associated with maintaining inventories in depots and include care and preservation expenditures, deterioration and obsolescence losses, interest on money invested, and others. Procurement costs are those associated with making a single procurement for a particular item and include costs incurred in supply management, industrial contracting and purchasing, district procurement activities, and fixed costs of manufacturing. More complete discussions of the nature and magnitudes of these costs are found in Section V of this report and in M.I.T. Interim Technical Reports Nos. 7 and 9.

It should be noted, particularly, that no costs have been associated with supply failures, although these costs are certainly substantial. Thus, if these proposals are adopted and national availability set higher than at present, the savings resulting from decreased expediting, and fewer shortage reports, extracts, back-orders, and explanations of poor performance will be a bonus over and above the savings claimed in this report.

2. Factors Affecting Costs and Inventories

The cost of procurement and cost of holding for a given item obviously directly affect the total costs of supply operation for that item. Besides the variance of demand, these are the only factors needed which are not presently used in supply control studies. Costs and inventories also increase as the lead time and variance-to-mean ratio increase. These two factors, as a matter of fact, have identical effects on costs and inventories in the mathematical model. A 50% increase (or decrease) in the lead time will increase (or decrease) total costs exactly as much as will a 50% increase (or decrease) in the variance-to-mean ratio. More will be said of this in Section IV. Finally, total operating costs and inventories depend upon the average demand rate and unit price.

3. Cost and Inventory Comparison

For purposes of comparing present and proposed procedures, an annual unit holding cost of 17% of unit price and a procurement cost of $100 plus fixed manufacturing cost have been assumed. The total cost and inventory figures shown here exclude all secondary items whose unit price exceeds $500 and all those whose annual demand rate is less than 100 units, as noted in Section II.

It is interesting to note that adoption of only the new safety level procedure at 99% availability would reduce stock-outs from 3% to 1% with almost negligible increases in costs and inventories. In other words, performance could be substantially improved by redistributing inventories, cutting some safety levels while raising others.

It is, perhaps, more interesting to note that cost savings generated by adopting the optimum system at 99% would quickly return the required additional investment in inventory. These savings are largely in the procurement area, and depend to a considerable degree on obtaining lower prices for larger orders.

EXHIBIT II

COST AND INVENTORY COMPARISON
(MILLIONS)

	HOLDING COSTS	PROCUREMENT COSTS	TOTAL OPERATING COSTS	AVERAGE WORKING INVENTORY
PRESENT SYSTEM				
BEFORE "5 X 5"	$18	$30	$48	$104
"5 X 5"	22	24	46	128
NOW (97 % AVAIL.)	25	24	49	150
PROPOSED SYSTEM				
90 % AVAIL.	$19	$13	$32	$113
95 % AVAIL.	22	14	36	131
99 % AVAIL.	27	15	42	162
99.87 % AVAIL.	32	15	47	191
PRESENT PROCUREMENT CYCLES, PROPOSED SAFETY LEVELS				
90 % AVAIL.	$17	$24	$41	$98
95 % AVAIL.	20	24	44	119
99 % AVAIL.	26	24	50	153
99.87 % AVAIL.	31	24	55	185

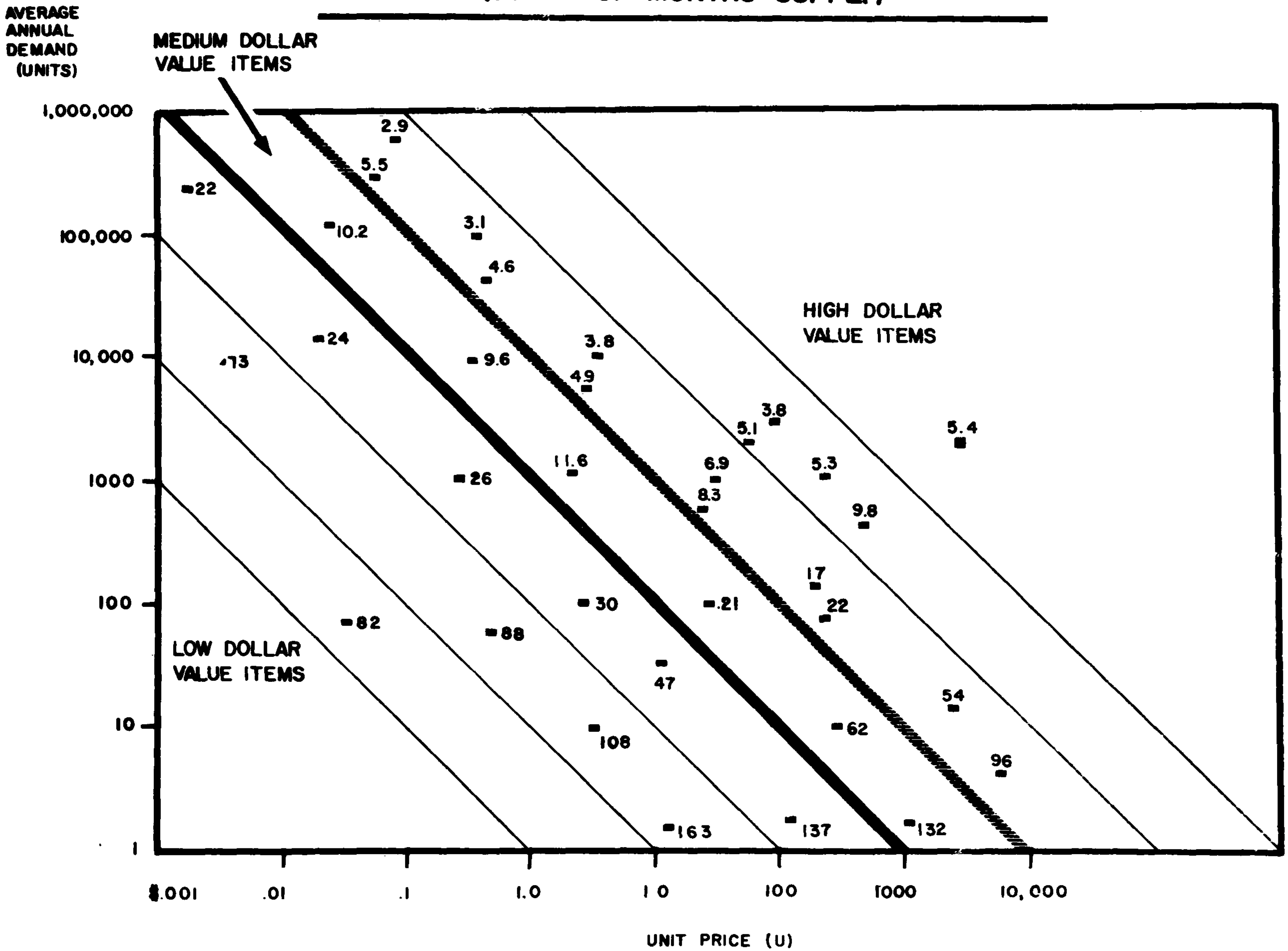
Procurement Cycles

The major impact of the proposed procedures on operating costs and inventories comes from substantial changes in procurement cycles. Exhibit 12 displays typical optimum procurement cycles for 99% availability as a joint function of price and demand rate. It must be remembered, however, that these cycles depend critically on the magnitude of the cost of procurement. For high-priced items this cost is assumed to be high, due mainly to the fixed cost of manufacturing, which should afford opportunities for substantial quantity discounts (see Exhibit 21, Appendix 1).

This exhibit once again demonstrates the hazards inherent in an approach which makes procurement cycles depend entirely on annual dollar demand. Exhibit 3, which showed average optimum procurement cycles as a function of annual dollar demand, is an oversimplification of the complex relationship seen here. By drawing lines between points of equal cycle length, one can see a pattern of contour lines emerging. These too, are averages, and may not be applicable to any particular item. They are properly used to study the relationships involved in this problem, and are not recommended for use in making supply decisions.

EXHIBIT 12

REPRESENTATIVE OPTIMUM PROCUREMENT CYCLES FOR 99 % AVAILABILITY (IN NO. OF MONTHS SUPPLY)



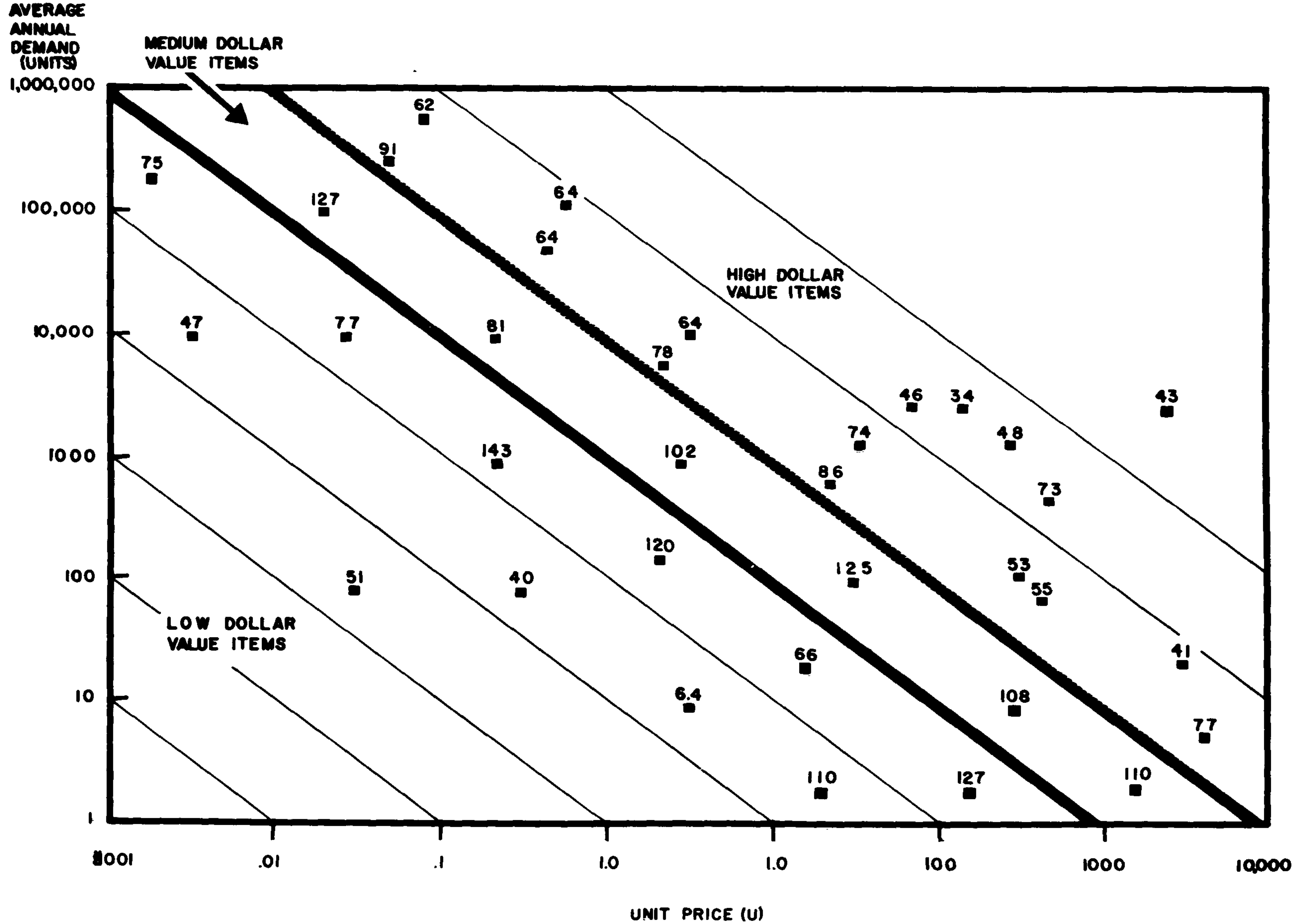
Safety Levels

Exhibit 13 shows typical optimum safety levels for 99% availability as a joint function of price and demand. These safety levels are based upon the assumption that optimum procurement cycles are used.

The fact that no consistent patterns are evident in this exhibit is further proof that levels should be established on the basis of each item's individual characteristics. The average relationship shown in Exhibit 2 hides much of the real problem.

EXHIBIT 13

REPRESENTATIVE OPTIMUM SAFETY LEVELS FOR 99% AVAILABILITY (IN NO. OF DAYS' SUPPLY)



SECTION IV

CRITICAL EVALUATION OF THE PROPOSED SYSTEM

This section of the report discusses those features of the proposed system that cannot be quantitatively defined or evaluated. Included is a discussion of the weak points of the proposed system, how they are treated under the present system and how, through further research studies, the proposed system may be further strengthened.

Forecasting of Average Demand Rates

The proposed system does not offer any mathematical aid to improving the forecasting of average demand rates. An error in forecasting the average demand rate will, as in the present system, lead to supply difficulties. The M.I.T. procedure will, however, reduce to a substantial degree the impact of under-estimation of future demand. This is because of the introduction of the concept of variance in the computation of the safety level and reorder warning point. Preliminary analysis indicates that the M.I.T. procedure can tolerate an error in the demand forecast of as much as 50% and still maintain the desired degree of protection against going out of stock during the procurement lead time. This factor alone is of great significance in comparing the procedures of the present and proposed systems.

This feature of the proposed system cannot, however, be substituted for knowledge, or considered judgements based on knowledge, of future conditions. The supply analyst must be kept continually aware of such influencing factors as anticipated troop activations or movements, changes in levels of Army operations, changes in major item densities, major item age, mutual security programs and rebuild programs, and he must use this knowledge to adjust demand forecasts. More emphasis should undoubtedly be placed on the consideration of demand rates computed by the Ordnance Supply Analysis Agency.

The research efforts of the Ordnance Supply Analysis Agency will most likely yield more information as time progresses on the effects of factors such as major item density changes, climate, tactical employment, etc. on demand rates. However, additional research work on forecasting techniques is also needed. Among the areas requiring immediate attention are methods for determining the extent to which demand history is useful for forecasting purposes, what lengths of past demand history, if it is used, should be considered in forecasting future demand rates, and methods for determining when a deviation from a forecast rate represents an actual change in demand rate rather than a random fluctuation.

The Assumption of Stationary Demand

The proposed system assumes that the average demand rate remains stationary (i.e., constant) during the period of forecast. While the present system permits adjustments to the average quarterly demand rate during the forecast period to accommodate the estimated effects of density changes, programmed demands, etc., there is abundant evidence from this and other studies, that random fluctuations are much more significant contributors to unsatisfactory supply performance than the assumption of stationary average demand. This does not mean, however, that the proposed system might not be further improved by modification to include provision for non-stationary demand rates during the forecast period. Consequently, additional work should be done to determine whether the additional benefits that might be achieved would justify the greater degree of mathematical effort that would be required.

Holding and Procurement Costs

It must be recognized that the proposed system requires that forecasts of these costs be made at the time of a supply review if the system is operated under the "least-cost" option. Under the present system, these estimates are not explicitly made; nevertheless, a good supply analyst does consider these factors, however intuitively, when he reaches his decision on how much to buy. One major advantage, then, of the proposed system is that these costs are explicitly recognized by the supply analyst. Further, it is anticipated that much improved bases for estimating these costs will become available as continuing data are generated through the Ordnance Command Management System.

Procurement Lead Time Variability

The proposed system assumes, as does the present system, that the forecast of procurement lead time will be met. Experience indicates that this is not always a reasonable assumption. There are, however, insufficient data available at this time to permit the treatment of procurement lead time in probabilistic terms. While this must be considered as a problem area requiring further study, it must be recognized that both the present and proposed systems are affected in the same way by errors in the procurement lead time forecast. The lead time forecasting problem is not as serious as the others, however, since there is provision under either the present or proposed systems for administrative action to expedite lagging deliveries.

Accuracy of Assets Information

The present system is frequently plagued by major inventory adjustments because of differences that develop between perpetual inventory records and physical inventory. These discrepancies will, of course, affect the performance under the proposed system in the same manner. It is anticipated that increasing utilization of Automatic Data Processing equipment will decrease these adjustments. In addition, a study is currently in progress to determine whether the application of statistical quality control techniques to the taking of physical inventories will improve this situation.

Consumer Funding

It is only recently that certain implications of the impact of consumer funding practices have been recognized. The most striking of these result from the practice of releasing large sums of consumer funds to continental and overseas armies at specific time intervals, usually at the beginning of each quarter. This has the effect of deferring demands for secondary items for a period of months, with a corresponding deluge of demands on the supply system when funds become available. A corollary effect has also been noted; it is apparently common practice for field armies to turn in serviceable parts which are sure to be needed in the near future in order to obtain credits with which to buy other parts which are in seriously short supply.

Neither the present nor the proposed system can be expected to operate effectively under these conditions. To the extent that consumer funding practices (or any others that are as yet undiscovered) tend to make demands experienced at the depot level less predictable, the effects to be expected are poorer supply performance, higher operating costs, and larger inventories. One immediately apparent technique for alleviating this difficulty would be to manage the funds in the same way as it is proposed to manage the parts--namely, to provide for a reorder point for funds replenishment that is consistent with anticipated spare parts demands.

Some mention should also be made of possible effects of implementation of the proposed system on Ordnance customers. One that can be looked for, even though it cannot be measured in a research study of this type, is customer reaction to improved supply performance. It has been often observed that demands for an item increase when it goes out of stock and that they decrease when stock again becomes available. It seems reasonable to assume that improvements in customer ordering habits will result from improved supply performance.

The Use of Mobilization Reserve Stocks

The actual performance of the present system is aided materially by the use of mobilization reserve stocks to satisfy peace-time operating requirements. It is entirely reasonable to do this, but the net effect is that there is no effective guarantee of having a minimum amount of stock on hand at the time of mobilization.

Adoption of the proposed procedures at, say, the 99% level of availability, would mean that stock-on-hand would be less than the mobilization reserve level only 1% of the time. Of course, if an item had no reserve stocks established, the above policy would mean the depot system would be out of stock 1% of the time.

Acceptance of the Proposed System by Supply Personnel

Questions have been raised at every discussion of the proposed system as to whether it can be readily used by today's supply analysts and whether it would require more time and more people.

There is no question but that careful training of supply analysts will be required before any such system is implemented. Nevertheless, the concepts underlying the system are simple and easily understood. No mathematical preparation is needed. Suggestions on procedures of implementation and training in their use are offered in the following section. It will be seen that little additional effort by the supply analyst is required except in the case of the more important costly items. It is in this area, however, that the need for more factual data and mature judgement is already recognized. The proposed system should be of great help to the analyst in enabling him to identify exactly those factors which should influence his decision and to determine quantitatively how each of these factors affects his decision. Moreover, the analyst, as he acquires experience in the use of the model, should be able to examine a family of decisions that are open to him and to estimate, in quantitative terms, the probable effects, in terms of both supply performance and costs, of each possible course of action. This type of analysis is not possible under the present system. Indeed, it is suspected that the present unavailability of analytical techniques (together with the enjoinder that they be used) is a source of frustration and irritation to today's supply analyst. There is no question that the magnitude of the supply problem demands better supply analysts. It is suggested that implementation of the proposed system, with utilization of its decision-aiding features, will help develop better supply analysts.

Achievement of Expected Performance Under Actual Operating Conditions

Throughout this report, emphasis has been placed on the fact that all comparisons and estimates have been based upon the policies and operating rules of the present and proposed systems. Examination of data on actual performance under present rules clearly shows, however, that actual results are not consistent with the theoretically determined estimates. One measure of this difference is shown in the number of procurement actions processed on stock fund items during Fiscal Year 1958. Theoretically, approximately 64,000 individual procurement actions should have been initiated. Actually, only 27,000 actions were processed. Similar differences are evident in the areas of supply performance where it is apparent that performance under present procedures in terms of national out-of-stock, percent of initial fill of requisitions and percent of on-time shipments should be better than it actually is.

Some of the factors that cause actual performance to deviate from what is theoretically possible under present procedures can be readily identified. The following are considered to be among the most important:

1. Reorder Warning Point Review

Theoretical estimates of supply performance (and for that matter, costs also) were based in this study upon a continuous review of reorder points--that is, the reorder point is checked each time an item is issued and replenishment action initiated immediately when the reorder point is reached. This performance is not now being achieved. Since the presently authorized safety level for most items is only thirty days' stock, any delay in discovering that a reorder point has been reached and in taking prompt replenishment action can greatly affect supply performance.

2. Forecasting Errors

Estimates of present system performance in this study were made on the basis of good forecasting. However, as has been shown earlier, the present system provides very little margin of protection against forecasting errors. Indeed, the present procedures seem to imply a measure of faith in the supply analyst's ability to foretell the future that is not at all supported by any known human capabilities. Consequently, it is not at all surprising that actual performance does deviate from the theoretically determined measures.

3. The "Dirty Data" Problem

In the theoretically determined performance measures, the assumption had to be made that data utilized in reaching decisions were timely and accurate. Some portion of the deviation of actual from theoretical performance is undoubtedly due to data errors in actual practice. Some of these would be missing, duplicated or incorrectly coded demand transactions, incorrect assets information, errors in Ordnance Master Data Files, etc. In actual practice, these errors would certainly account to some degree for forecasting errors, improper procurement decisions, missed shipment dates and the like.

4. The Human Element

The theoretical estimates of present system performance assume that a supply analyst, given a set of facts, will reach a supply decision that is consistent with the rules and, further, that all analysts faced with the same facts will reach identical decisions. It is, of course, recognized that no set of rules can be so clearly stated that this is possible of achievement. Moreover, it is recognized that no two supply analysts possess to an identical degree the level of skill, knowledge and motivation necessary to follow exactly even the most precisely stated instructions. Thus, it can be expected that any theoretical estimates of performance will differ from results achieved under actual operating conditions.

It would, of course, be nonsensical to assume that adoption of the proposed system would automatically solve difficulties of the types enumerated above. It can be stated, however, that adoption of the M.I.T. procedures would improve supply performance to a marked degree even if no improvement were made on these factors. This statement can be made with a high degree of confidence simply because the provision for random fluctuations in safety levels would afford a greater degree of protection from vagaries of this kind than can possibly be obtained under the present rules. The only condition that must be placed on this statement is that theoretically possible performance under the proposed system will not be achieved to the degree indicated by the estimates so long as the above difficulties exist. In other words, while actual performance under the M.I.T. procedures can be confidently expected to be significantly better than under present procedures, the actual quantitative measure of improvement may turn out to be somewhat less than indicated by the theoretical estimates.

There is, however, a real basis for optimism in feeling that the theoretically available improvements will be very nearly possible in the foreseeable future. Wider application and increasing skill in the utilization of Automatic Data Processing Systems should do much in improving supply system performance. The continuous check of reorder warning points, for example, is easily practicable with a computer. Similarly, data errors are much more easily detected, and they can be detected while the error is fresh and readily correctible. In addition, as ADP Systems take over more and more of the routine clerical tasks of supply personnel, more time will be made available for the non-routine decision-making that is so important to the success of any system. Moreover, as has been contended previously, the M.I.T. procedures by their very structure will provide a much sounder basis for the supply analyst's decisions than is now available and should contribute to increasing his skill and knowledge.

A Base for Further Improvements

The proposed system provides for an explicit and logically consistent treatment of all the major factors which affect the performance, operating costs, and inventory investment of a depot supply system. As such, the proposed system will be invaluable as a base for further improvements by supplying operating experience and data.

A number of possibilities are apparent. It will come as no surprise to anyone familiar with military procurement problems that simplification of procurement procedures, reduction of procurement lead times, and smoothing of demand patterns would improve performance and cut costs as well.

It has not always been possible, however, to predict the effects of specific changes designed to accomplish these ends. The mathematical model underlying these recommendations, and the computer simulation program designed to test it can make such predictions much more accurate and reliable.

A particular example will serve as illustration. It has been found that, if the variance of demand could be cut to its theoretical minimum, operating costs and inventories at 99% availability would be roughly half of what are now required. To reduce the variance to this level would require the following:

(1) Customers arrive at the depots in a random fashion (Poisson Distribution).

(2) All orders are for a quantity of one.

This extreme is, of course, out of the question. The paperwork involved would be prohibitive, but it is perhaps possible of achievement for selected high dollar items and this is where most of the total costs and inventories lie.

There are obviously difficult procedural problems here. However, the possibility of a 50% reduction in many high dollar item operating costs and inventories is not to be lightly dismissed. This illustration leads also into the whole question of the depot-customer relationship.

One of the paradoxes uncovered by the study indicates that economical order quantities for posts, camps and stations are not so simply determined as had been thought. Paperwork and transportation costs are reduced by large orders, but the variance of depot demand is thereby increased since variance is roughly proportional to average size of orders. And variance is very expensive as we have seen.

No attempt has been made to solve the general problem of optimum ordering procedures for the posts. It would certainly be fruitful to base a study in this area on the national system model already developed.

SECTION V

SUGGESTED IMPLEMENTATION PROCEDURES

Revision of AR 710-45

This Army Regulation prescribes the supply control procedures to be followed for minor secondary items and repair parts. Very little revision of this AR would be required to permit adoption of the M.I.T. procedures since the AR already recognizes that safety levels may be subject to change and that maximum procurement limitations may be waived when suitable justification exists. These, therefore, are the only revisions that would be necessary:

1. Reorder Point, Paragraph 9c, Page 13

Add the following ... "The safety level component of the reorder point may be computed on an individual item basis, regardless of dollar value classification of the item, where it is apparent that unpredictable fluctuations in supply demand are of such magnitude as to require special treatment. In such cases, the variance of the unpredictable fluctuation may be taken into account."

2. Exceptions and Uneconomical Procurement Quantities, Paragraph 10b(4), Pages 17, 18

Add the following ... "Prior approval for exceptions need not be requested from the Deputy Chief of Staff for Logistics if heads of technical services have implemented a formal and quantitative procedure for determining the economical buy. In all cases, however, where the procurement quantity deviates from the procurement quantity computed according to instructions herein, the relevant cost factors and methods of their consideration must be documented and made a part of the formal supply control study."

Estimation of Factors

If only the safety level feature of the M.I.T. model is implemented, the sole new piece of information required is the variance of demand in the lead time for each item. If least-cost operation is desired, the M.I.T. procurement cycle calculation requires that the costs of procurement and holding be known.

1. Variance of Demand

Two methods of obtaining the variance of demand are available, one involving manual or computer use of a nomograph, the other requiring computer or EAM analysis of demand data. Which of these is the better predictor of variance is a moot point, and work is being continued in this area. It seems desirable, however, to use the nomograph method for actual supply decisions since it is much the simpler of the two.

The nomograph found in Exhibit 15, was constructed by analyzing data on about 150 Ordnance secondary items (See Appendix I).

2. Costs of Procurement and Holding

AR 710-45 already recognizes the concept of and necessity for economical procurement quantities. If economy of operations is to be achieved, Supply Management personnel must know the costs of doing business and must become accustomed to thinking in these terms. In principle, the costs of procurement and holding are different for each item. In practice, the differences are negligible for large classes of items. Therefore, it is suggested that the cost of procurement be taken as $100 plus fixed manufacturing cost and the cost of holding as 17% of unit price per year, with deviations permitted when substantiated.

Before examining in detail the composition of these costs, it would be well to look at the principles underlying the proper estimation of costs to be used in making decisions. Foremost is the following: No cost is relevant in making a decision unless that cost will be affected by the decision. This means, for example, that in making routine procurement decisions most overhead costs are irrelevant. It also means that even potential savings in direct labor are illusory unless people can be discharged, transferred, or enabled to do their present jobs appreciably better.

a. Procurement Costs

The administrative costs of procurement are largely direct labor costs at national control points and district procurement offices. A procurement cost study of substantial size has been done and is included in Interim Technical Report No. 9. Work is being continued in this area by the Cost Committee of the Ordnance Inventory Management Project.

There is another part of the cost of procurement which has received somewhat less attention, but is most important. Any parts of the bidders' final quotations which do not depend upon the quantity being procured, i.e., fixed manufacturing costs, are also costs of procurement. They are paid as often as new procurement necessitating a new manufacturing run is made, and can amount to thousands of dollars for some items. This is, of course, the concept underlying quantity discounts. Methods of estimating these costs are available, the most sensible and direct of which would consist in asking for bids on more than one quantity. Engineering estimates could be used provided one could make the assumption that contractors would bid on the same basis.

b. Holding Costs

As in the case of procurement costs, a detailed study of holding costs has been made, and is reported in Interim Technical Report No. 9. Exhibit 14 summarizes the results of this study. Supply Control Point estimates of deterioration and obsolescence rates ranged from 0% to 14% and 0% to 10%, respectively.

The relative range of holding costs is much less than that of procurement costs. The latter, including administrative and fixed manufacturing costs, can range from less than $100 to many thousands of dollars. Holding costs, on the other hand, will almost always be found to be between 10% and 25% per year except for those few items subject to extremely high deterioration or surprise obsolescence rates. As long as these exceptional items are recognized and handled accordingly the use of an average rate of 17% per year should yield excellent results. An error of a factor of two (50% under or 100% over) in estimating either holding or procurement costs, will result in an increase in total costs of only 5.6%.

Exhibit 14

Summary of Holding Cost Data Used in Simulations

Element Recognized	Source Used	Method of Calculating Annual % per $ per item
Maintenance in storage	E = DCMS 2220 accounts .1230 + .1330, calendar 1957	$\frac{E}{V}$ for an average item, adjusted depending on nature of item, and ranging for the 24 items from .4% to 2%
Losses	O.C.O. advice	2%, as in stock fund accounting
Additional Deterioration	Commodity Command estimates	Life-time, or time until restoration; converted to short-term % rates depending upon whether initial deterioration was deemed to increase linearly or exponentially.
Obsolescence	Commodity Command estimates	Either linear or exponential % rate, depending upon technological nature of item
Investment	O.C.O. advice	3.5%, all items

Detailed Computational Method

The computational methods involve only ordinary arithmetic and three nomographs (two if the least-cost procurement cycle feature is not used). Thus these calculations can be performed either by present supply analysts or by electronic computer.

1. Variance Calculation

The nomograph for determining the variance-to-mean ratio V is found in Exhibit 15. Work is continuing on the problem of finding the best fit of V plotted against price and demand. Thus, it is possible that these contour lines may shift slightly as a result of additional calculations.

To illustrate the use of this and succeeding nomographs, let us consider a specific example, a fictitious item called a widget with the following characteristics:

annual demand	=	λ	=	200 units
unit price	=	u	=	\$40.
variance-to-mean ratio	=	V	=	13 (from Exhibit 15)
holding cost	=	i	=	.17 per year
procurement cost	=	C_p	=	\$900 (See Exhibit 21)
procurement lead time	=	L	=	.67 year

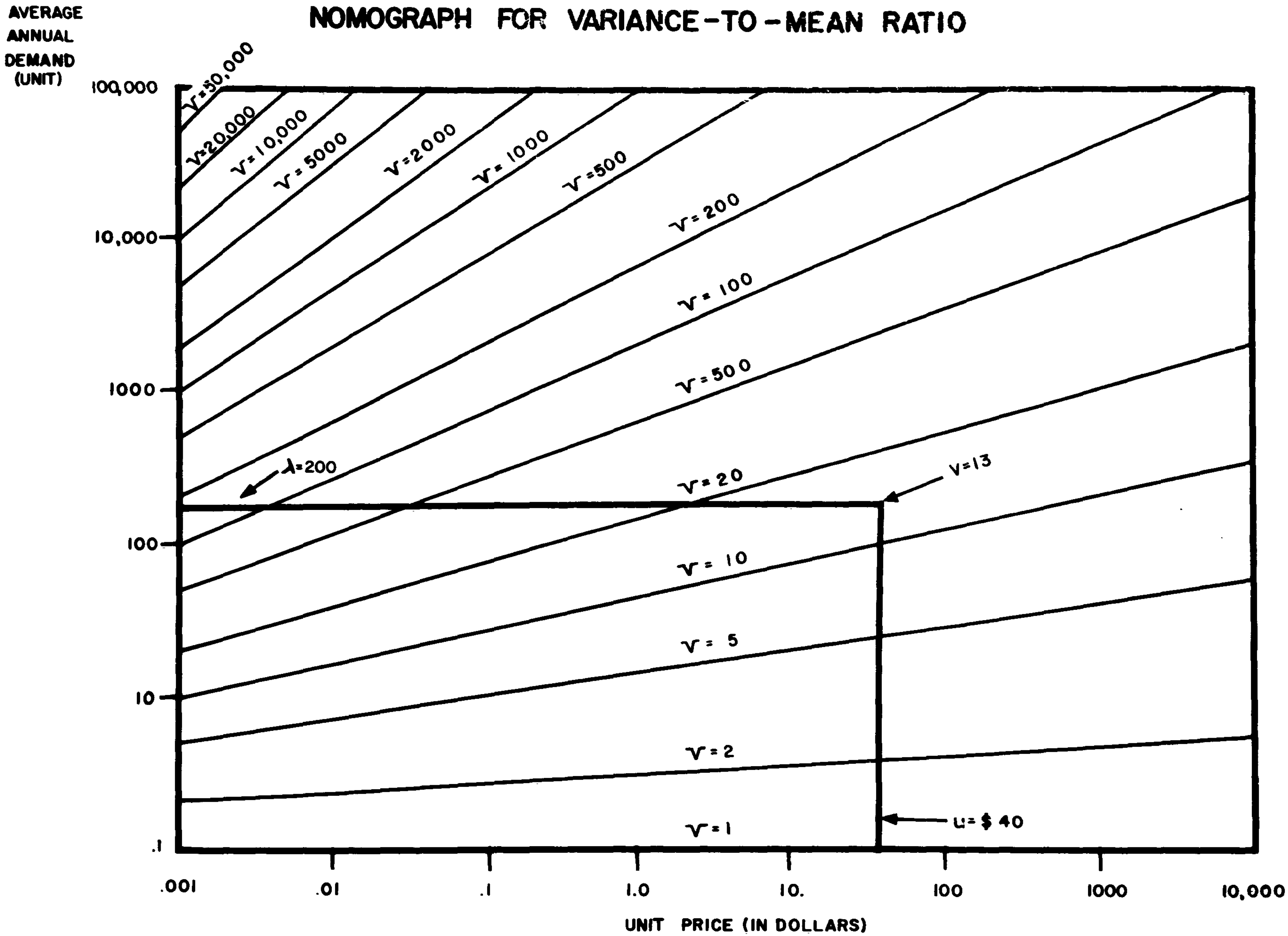
After obtaining the variance-to-mean ratio (V = 13) as shown in Exhibit 15, one must calculate the standard deviation of demand σ_L.

$$\sigma_L = \sqrt{\lambda LV} = \sqrt{(200)(.67)(13)} = 41.7$$

Then one proceeds to the calculation of the procurement cycle level.

EXHIBIT 15

NOMOGRAPH FOR VARIANCE-TO-MEAN RATIO



2. Procurement Cycle Level Calculation

To calculate the optimum procurement cycle level, one follows the procedure in Exhibit 16.

Continuing with the example from the variance calculation, one finds that

$$Z = \frac{\lambda\, C_p}{i\, u\, \sigma_L^2}^{*} = \frac{C_p}{i\, u\, V\, L} = \frac{\$900}{(.17)(\$40)(13)(.67)} = 15.2$$

For 99% availability, this corresponds to b = 5.82, as shown in Exhibit 16. (A precise Z, b nomograph will be found in Exhibit 24.) The procurement cycle level Q is given by

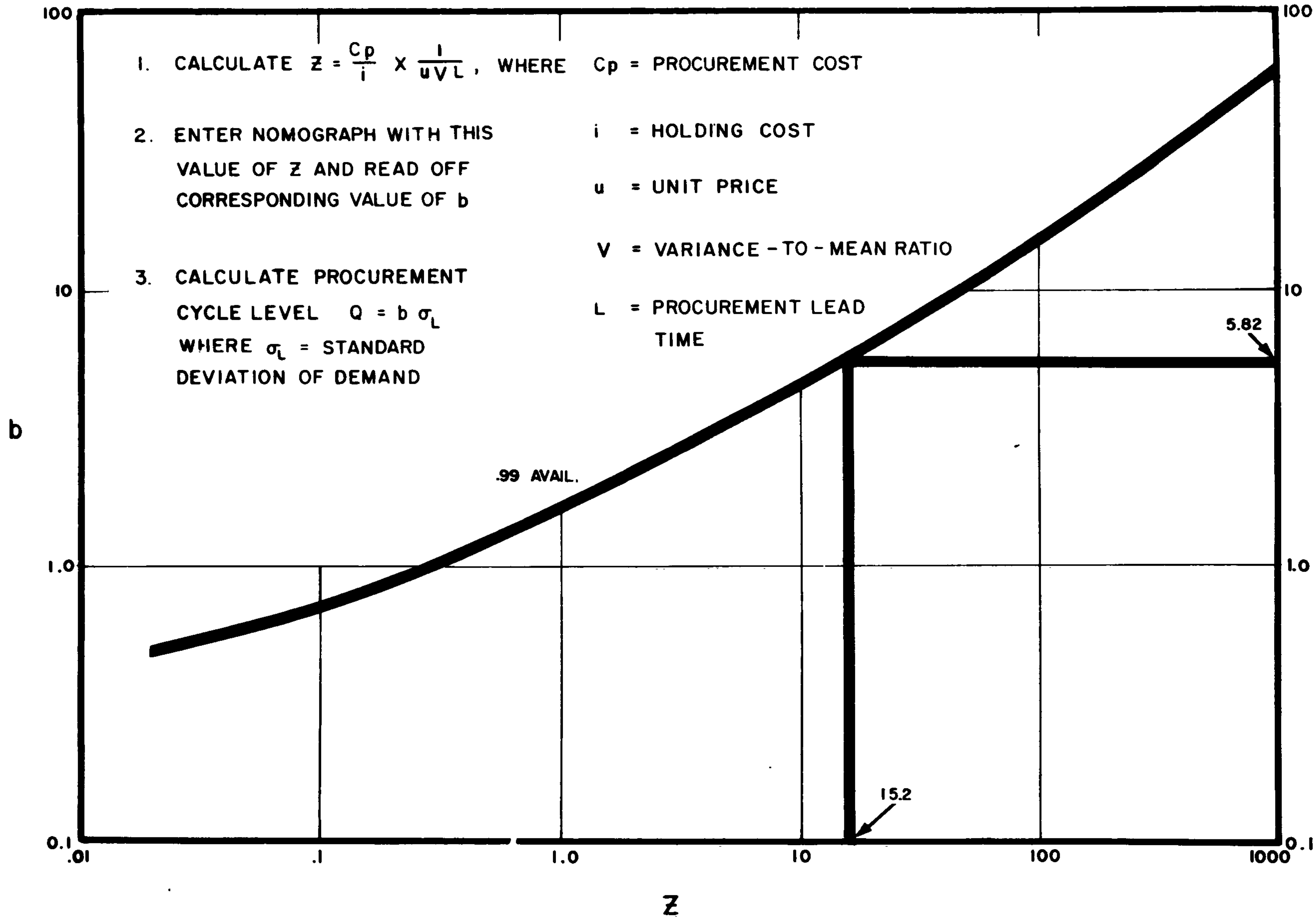
$Q = b\, \sigma_L = (5.82)\ (41.7) = 243$ units, σ_L being taken from the previous calculation.

This value of Q, the optimum procurement cycle level for the widget, amounts to 14.6 month's supply. The present method of calculating Q would yield a level of 6 months' supply, 100 units, since the widget is a medium dollar value item ($\lambda\, u$ = $8000 per year).

* Those familiar with the Wilson formula for optimum procurement quantities will recognize that $Z = \frac{Q_W^2}{2\, \sigma_L^2}$, where Q_W is the Wilson lot size and σ_L^2 is the variance of demand in the lead time.

EXHIBIT 16

CALCULATION OF LEAST COST PROCUREMENT CYCLE LEVEL



3. Safety Level Calculation

In order to calculate the safety level, one must first specify the procurement cycle level. But, it is not necessary to use the preceding method. Any procurement cycle level for any item has a unique safety level associated with it which will provide a specified level of availability.

Given the procurement cycle level, the desired availability, and the standard deviation of demand (from the variance calculation), one follows the procedure set forth in Exhibit 17 to find the proper safety level. (A precise a, b, nomograph can be found in Exhibit 25.)

The reorder point is then the sum of the procurable mobilization reserve material requirements, the expected demand in the lead time, and the safety level.

$$R = (\text{PMRMR}) + \lambda L + a\sigma_L$$

If one wishes to use the optimum procurement cycle level, the quantity b is already known to be 5.82 from the previous calculation. Then, a = 1.17 as shown in Exhibit 17 if an availability of 99% is specified. The safety level is given by $a\sigma_L = (1.17)(41.7) = 49$ units, about 89 days' supply.

Under present regulations, the safety level would be a 60 days' supply. With a procurement cycle of 6 months, this safety level would result in an average national availability of 95% for the widget.

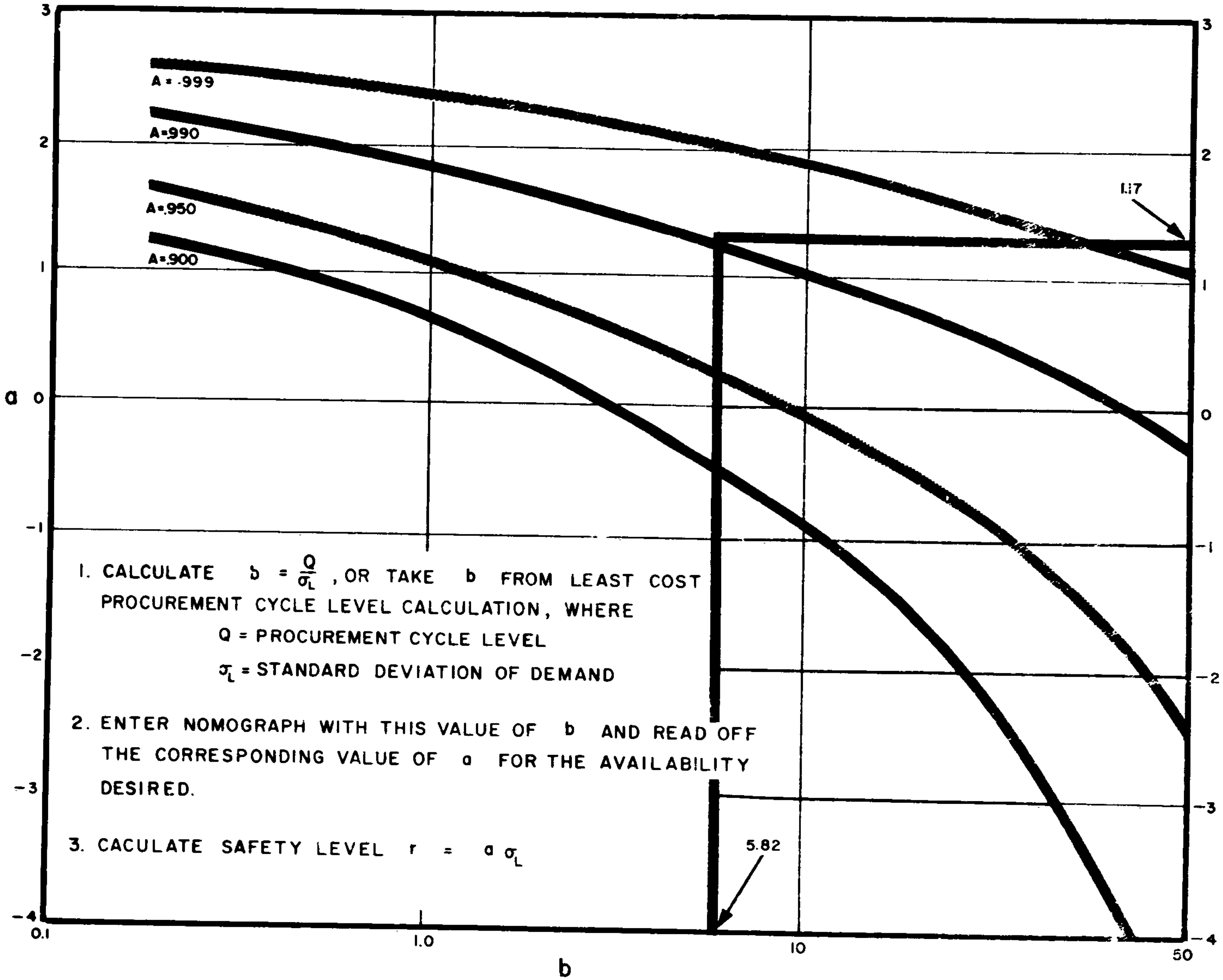
If one wanted to use the present 6 month procurement cycle and still achieve a 99% availability, the following calculation gives the necessary safety level.

$$b = \frac{Q}{\sigma_L} = \frac{100 \text{ units}}{41.7 \text{ units}} = 2.40$$

From Exhibit 17, a = 1.58, so the safety level is $a\sigma_L = (1.58)(41.7) = 66$ units or 120 days' supply.

EXHIBIT 17

CALCULATION OF SAFETY LEVEL



4. Summary of Example

	Present Q, Present R	Present Q, Proposed R		Proposed Q Proposed R	
Availability	95%	95%	99%	95%	99%
Procurement Cycle (Mos.)	6	6	6	15.6	14.6
Safety Level (Days)	60	60	120	14	89
Average Working Inventory[1]	$3330	$3330	$4640	$5510	$6810
Average Annual Operating Costs[2]	$2366	$2366	$2588	$1628	$1899

---

1. Average working inventory (I) is given by the equation $I = u\left(\frac{Q}{2} + a\sigma_L\right)$ to a very close approximation for high levels of availability.

2. Average annual operating costs (O.C.) are given by the equation $O.C. = i\,I + \frac{\lambda C_p}{Q}$, the term $i\,I$ being average annual holding costs and $\frac{\lambda C_p}{Q}$ average annual procurement costs (number of procurements x cost of procurement).

Machine Procedures

While the M.I.T. procedures can be implemented on a manual basis, it is obviously desirable to make maximum use of EAM and ADP equipment. The following modes of operation are therefore suggested:

1. Low Dollar Value Items

a. With ADPS

The complete supply control study can be performed within the computer. Computation of an average annual demand rate, taking into consideration both recurring and nonrecurring demands, can be done in the computer by analysis of historical demand. This rate, together with the unit price of the item, is sufficient for the computer to determine the variance-to-mean ratio and safety level by table look-up in an internally stored nomograph. The reorder point would then be internally computed and stored, other levels computed, assets applied and procurement or excess determination made in much the same manner as under present machine procedures. If implementation of the least-cost mode of operation is desired, the computer can determine the optimal quantity to be procured (or otherwise obtained nationally) by applying standard procurement and holding cost estimates. From data obtained during the course of this study, a procurement cost of $100 and a holding cost of 17% appear most reasonable for low dollar value items.

The supply analyst should review the machine study to determine whether any revision is necessary. For example, he might have knowledge of special conditions that would require a change in the demand forecast computed by the machine or he might want to change the values of the holding or procurement costs because of special characteristics of the item. If this is the case, changes can be made by reference to the nomographs in the manner described in the section on computational methods.

b. With EAM Equipment

A supply control study cannot be accomplished on a completely mechanical basis with EAM equipment because of practical difficulties in handling tables of nomograph values. Under the procedure suggested below, however, the amount of manual intervention required would be relatively small and should add only a few minutes to the manual work that is required on EAM low dollar value studies under current practices.

It is feasible to utilize EAM equipment to compute the average demand forecast from demand data in the manner described in paragraph 1.a., above. The demand rate thus computed would then be furnished to the analyst for review and revision where necessary. At this point, he would obtain the variance from the nomograph (Exhibit 15) and calculate the safety level. The safety level quantity would then be punched on an EAM card and the balance of the supply control studies performed on EAM equipment in much the same way as it is presently done.

If least-cost operation is desired, the analyst would also have to determine the optimal procurement cycle quantity in the manner described in the section on computational methods. This would be done at the time the safety level is computed. The balance of the study could then be done by EAM equipment.

2. High and Medium Dollar Value Items

a. With ADPS

It is probable that only those installations with ADP equipment will be able to mechanize high and medium dollar supply control studies to any appreciable extent. The basic differences in approach that might be suggested in the case of high and medium dollar value ADPS studies concern the demand forecast and, if least-cost operation is desired, the estimates of procurement and holding costs. In high and medium dollar items, it is probably desirable to have the analyst examine demand data carefully before arriving at a demand forecast. Under the least-cost mode of operation, the analyst should also carefully review the most recent procurement and holding cost estimates to determine whether changes should be made. However, once the estimates of procurement and holding costs and the demand forecast are arrived at, the balance of the study should be feasible on ADP equipment. The completed study would naturally be reviewed and revisions made, where necessary, by use of the nomographs.

b. With EAM

EAM equipment can, of course, be used to assemble and summarize demand and asset information for the analyst. From here on, the procedure would probably have to be manual, as it is now. The additional steps required would be the determination of the safety level by the M.I.T. method as described above, and, if least-cost operation is desired, the determination of the optimal procurement cycle quantity.

Procedural Instructions

As recommended at the Ordnance Supply Operations Conference in January, 1959, it is suggested that procedural instructions for implementing the proposed system be drafted by the Ordnance Field Systems Agency with the assistance of supply analyst personnel from the National Inventory Control Points. Assistance by personnel of the Ordnance Inventory Management Project and M.I.T. would also be available. These procedural instructions should be written as revisions and appendices to ORDM 3-2 and ORDM 3-6. Instructions, particularly in ORDM 3-6, should include illustrative material on the application of the model under a variety of circumstances and sample supply control studies done under both the present and proposed systems.

As part of the procedural instructions package, there should also be developed new supply control study forms, preferably simpler than those in use today. EAM card alignments should also be developed for input data needed for supply control study computations and detailed EAM procedures might also be developed for those installations not having ADPS equipment.

Training

The amount of training required will, of course, vary with each supply analyst. It is believed that 8-10 hours of formal instruction time should suffice to teach the fundamental principles of the proposed system. This could be done by the NICP representatives who participate in writing the procedural instructions. Proficiency in use of the system from that point on should come from continued use of the system in actual work situations.

APPENDIX I

METHODS OF OBTAINING COST AND PERFORMANCE ESTIMATES FOR PRESENT AND PROPOSED SYSTEMS

Item Characteristics

In addition to national availability, there are for each secondary item 6 basic elements which must be measured in order to utilize the mathematical formulae for availability, optimum procurement cycles and associated operating costs and inventories. They are:

(1) Annual demand rate, λ

(2) Approximate unit price, u, in dollars

(3) Lead time, L

(4) Estimated variance to mean ratio, V (measure of short-term fluctuation in demand)

(5) Variable procurement costs C_p (administrative in Ordnance plus equivalent of manufacturing setup cost and/or quantity discount)

(6) Variable cost of stocking, i, (variable with average amount stocked) in ¢ per dollar stocked per year.

In addition, of course, the desired national availability is assumed to have been specified.

The following data on elements (1) - (6) above were gathered and used in the study.

1. Annual Demand and Unit Price, (λ and u)

For the purpose of calculating the effect of implementing the procedures on secondary items generally, or on any selected group, it is necessary to know the percentage distribution of secondary items in each category jointly of annual demand rate and unit price. More particularly, since not all items are repetitively procured (for some items, issues are largely met out of excess inventories; others are becoming obsolescent), the basic problem is to identify a reasonable selection of secondary items which are in fact being repetitively procured.

Two main sources of information were available on demand rate and unit price (1) the OSAA-6 list (2) activity reports from NICP's on stock-fund items.

A joint distribution of λ and u was run in January 1959 on the IBM 704 at M.I.T., from individual records on each of approximately 46,000 secondary items on the OSAA-6 list. This demand is of course recurring demand only. The results are shown in Exhibit 18.

EXHIBIT 18

JOINT DISTRIBUTION OF DEMAND AND PRICE

(OSAA #6 Items)

	ANNUAL QUANTITY DEMANDED (UNITS)							
PRICE	0	1-9	10-99	100-999	1,000-9,999	10,000-99,000	>100,000	TOTAL
$ 0 - .01	2.661%	0.150%	0.371%	0.882%	0.790%	0.161%	0.002%	5.017%
.01 - .10	7.101	0.698	1.678	2.160	1.008	0.160	0.022	12.827
.10 - 1.0	15.182	2.926	4.424	3.442	1.589	0.285	0.019	27.867
1.0 - 10	19.571	4.204	5.196	3.051	1.003	0.134	0.005	33.164
10 - 100	10.001	2.160	2.316	1.181	0.279	0.039	—	15.976
100 - 1,000	2.677	0.796	0.619	0.212	0.071	—	—	4.375
1,000 - 10,000	0.427	0.144	0.135	0.017	0.023	—	—	0.746
> 10,000	0.020	0.002	0.004	0.002	—	—	—	0.028
TOTAL	57.640%	11.080%	14.743%	10.947%	4.763%	0.779%	0.048%	100.000%

This distribution was combined with the reports in (2) above to yield a rough estimate of λ, u, and the number N (on the basis of 150,000 stock fund items) for total demand recurring and nonrecurring. The results are shown in Exhibit 19.

EXHIBIT 19

JOINT DISTRIBUTION OF DOLLAR DEMAND AND UNIT PRICE

(150,000 Stock Fund Items)

Legend: u = unit price in $
n_u = estimated number of items having the price (from OSAA #6 analysis)
N = estimated number of items (in thousands) having an annual dollar demand λu
B = estimated annual business (dollar demand) in millions of dollars*

		ANNUAL DOLLAR DEMAND ($000)											
		< .1		.1 - 1		1 - 10		10 - 25		25 - 50		> 50	
U	n_u (000)	N	B	N	B	N	B	N	B	N	B	N	B
> $10,000	.03	–	–	–	–	–	–	–	–	–	–	.03	
1,000 - 10,000	1.1	–	–	–	–	0.1	0.5	0.4	7.0	0.2	8.0	0.4	
100 - 1,000	6.0	–	–	1.0	0.4	4.0	12.0	0.4	8.0	0.3	12.0	0.3	
10 - 100	21.4	6.0	0.2	8.0	3.2	6.3	20.0	0.6	11.0	0.3	12.0	0.2	
1 - 10	48.47	13.6	0.4	30.0	12.0	4.3	14.0	0.4	8.0	0.1	4.0	0.07	
0.1 - 1.0	45.05	36.4	1.2	6.0	2.4	2.2	7.0	0.3	5.0	0.15	6.0	0.0	
0.01- 0.1	19.95	16.0	0.5	3.0	1.2	0.8	2.5	0.1	1.0	0.05	2.0	0.0	
< .01	8.0	6.0	0.2	2.0	0.8	–	–	–	–	–	–	–	
TOTAL	150.0	78.0	$2.5	50.0	$20	17.7	$56.0	2.2	$40.0	1.1	$44.0	1.0	$337.5

*Based on total annual business of $500 million.

2. Lead Time

A plot of procurement lead times reported by the several NICP's on 71 secondary items, selected at random, is presented in Exhibit 20.

3. Variance to Mean Ratio, (V)

In February 1959, extensive analysis was made at M.I.T. of historical records of individual daily depot demand transactions for 53 secondary items. Of these, approximately 20 were OTAC items on which 6 months' continuous history (1957) was available. The balance were Frankford items, on which 34 months' continuous history was available. The data was of each demand individually as to date, quantity, customer, transaction analysis, etc.

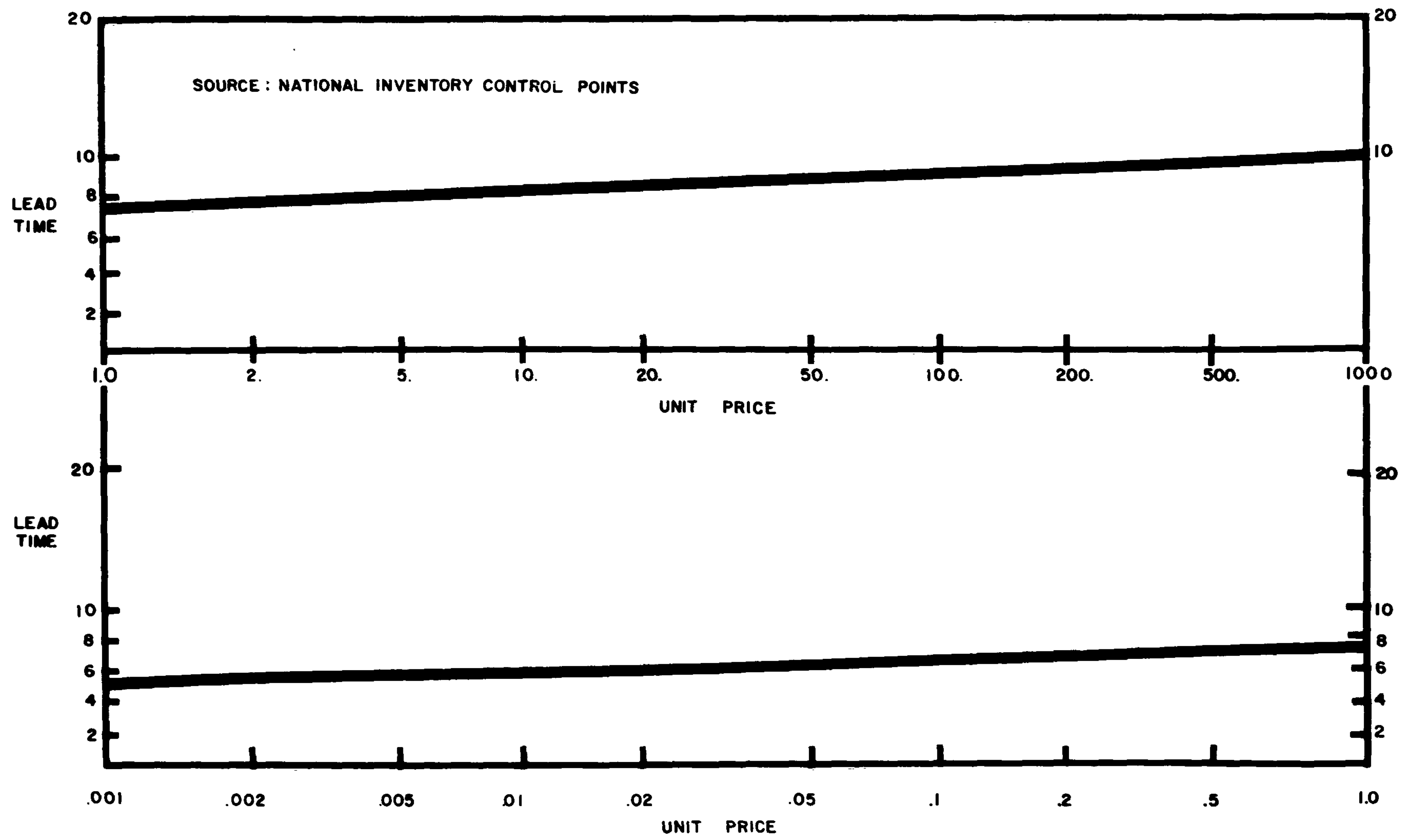
In addition to much information of incidental interest being revealed (such as the difference in affect upon demand fluctuation between orders from overseas and those from continental customers), the resulting measures of V showed that demand rate and unit price play a large role in determining the value of V which may be expected on an individual secondary item.

The observed values of V were used to establish the average values given in Exhibit 15 (page 55).

It will be noted that V is there revealed as (1) decreasing (towards the value 1 as a limit) as unit price increases (at a fixed annual demand rate) and (2) increasing, at a fixed unit price level, as annual demand rate increases. This variation expresses not only the property that expensive items are ordered in smaller quantities and more in relation to immediate real usage, than inexpensive ones, but also simultaneously expresses the fact that such discipline is harder to exercise on fast-moving than on slow-moving items.

EXHIBIT 20

PROCUREMENT LEAD TIME (IN MONTHS) VERSUS UNIT PRICE (IN DOLLARS) (71 SECONDARY ITEMS)



4. Variable Procurement Costs

The several supply control points furnished estimates of manufacturing setup cost on a total of approximately 150 items.

The regression line shown in Exhibit 21 was obtained by averaging the square roots of $100 + the value of manufacturing setup cost shown in the figure. This average was then used in the final estimates described in Section III, below.

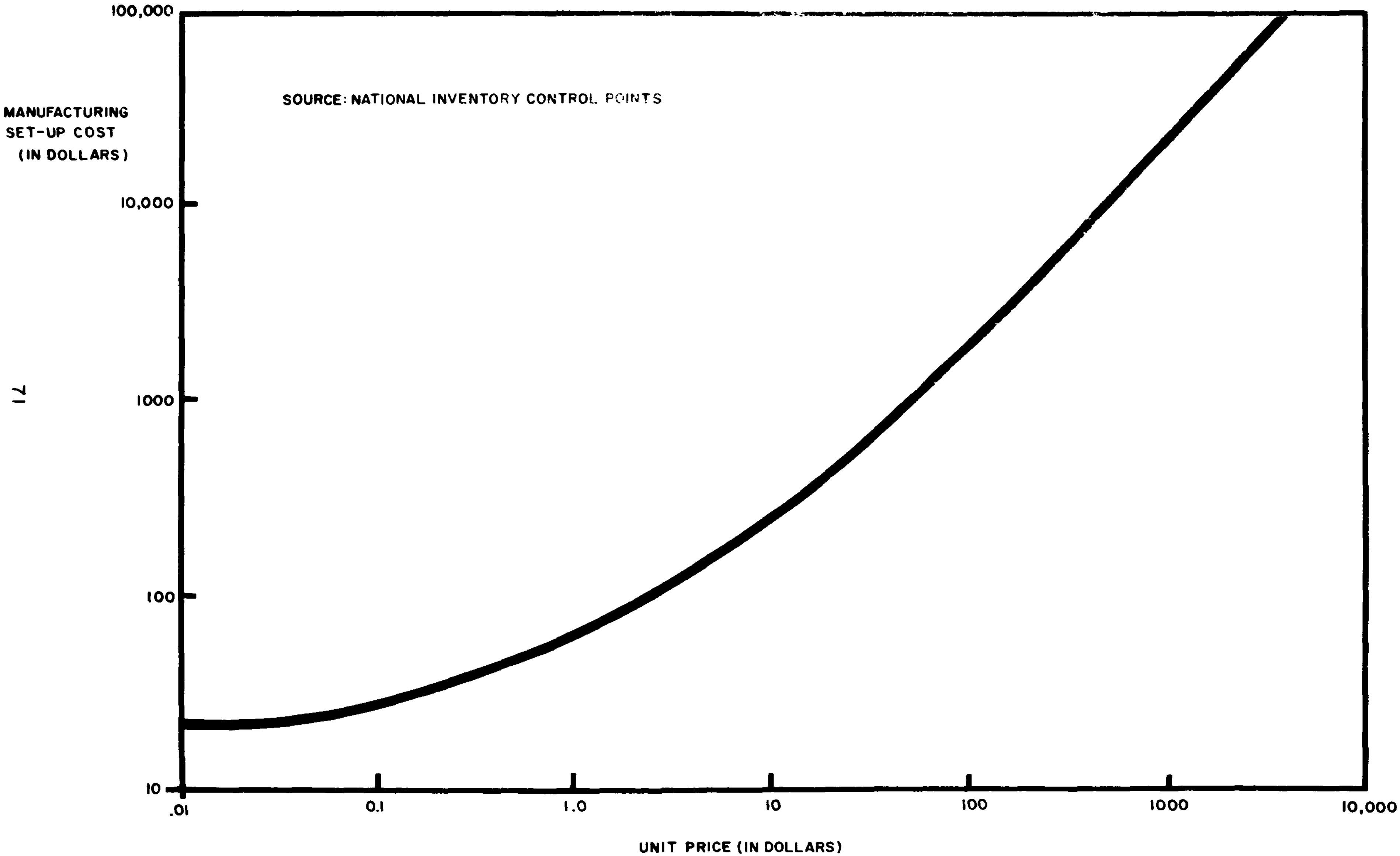
It is to be emphasized that this average is not necessarily the value to be used on any individual secondary item under implementation of these procedures. The departure of an item from this average plays a critical role in determining the proper procurement cycle.

5. Variable Cost of Stocking

For the purposes of estimates, a standard cost was used of 17% annually per dollar of average inventory.

EXHIBIT 21

AVERAGE MANUFACTURING SET-UP COSTS



Cost, Inventory, and Availability Calculation

In order to assess the total impact of the proposed system on costs, inventories, and performance, a sample of 34 typical items was used. These are shown in Exhibit 22. The values of L, C_p, and V were taken from Exhibits 20, 21, and 15, respectively.

For each of these 34 items calculations were made of costs, inventories, and availabilities under the various procedures for determining the safety levels and procurement cycles. The appropriate formulae are all found in Appendix II. An auxiliary nomograph was used in the case of optimum procurement cycles for determining the expected value of inventory divided by the standard deviation of demand in the lead time. This is shown in Exhibit 23.

The individual estimates of costs, inventories, and availabilities were multiplied by N, the number of items in a particular demand and price category, and the results summed in order to obtain total estimates.

EXHIBIT 22

ITEM CHARACTERISTICS FOR 34 ITEM SAMPLE

	Range of λu (000)	Unit Price u	Annual Demand (Units) λ	Number of Such Items N,000	Annual \$ Business = $N\lambda u$ in Millions	Years L	Cp
1	\$ <.1	\$ 20	1.66	6	.2	.717	\$ 500
2		3	10	13.6	.4	.658	209
3		.5	66	36.4	1.2	.612	146
4		.04	780	16	.5	.533	120
5		.003	11,000	6	.2	.458	100
6	.1-1	200	2	1	.4	.787	4,150
7		20	20	8	3.2	.717	500
8		2.5	160	30	12.	.654	195
9		.4	1,000	6	2.4	.604	145
10		.03	13,000	3	1.2	.529	120
11		.002	200,000	2	.8	.442	100
12	1-10	2000	2.5	.1	.5	.842	49,100
13		300	10	4	12	.800	6,500
14		31	100	6.3	20	.729	700
15		3.25	1,000	4.3	14	.662	214
16		.32	10,000	2.2	7	.600	139
17		.031	100,000	.8	2.5	.529	122
18	10-25	3500	5	.4	7	.850	90,100
19		250	80	.4	8	.792	5,300
20		30	610	.6	11	.729	680
21		2.5	8,000	.4	8	.654	199
22		.4	41,500	.3	5	.604	145
23		.05	200,000	.1	1	.542	122
24	25-50	2500	16	.2	8	.846	62,100
25		300	135	.3	12	.800	6,500
26		40	1,000	.3	12	.733	880
27		4	10,000	.1	4	.671	228
28		.4	100,000	.15	6	.604	145
29		.08	500,000	.05	2	.558	124
30	50-100	70	1,670	.45	50	.758	1,450
31	100-250	100	3,000	.20	60	.767	2,100
32	250-500	200	1,560	.16	50	.787	4,150
33	500-1,000	500	470	.15	40	.817	11,100
34	>1,000	2000	1,725	.04	138.5	.842	49,100

EXHIBIT 23

INVENTORY NOMOGRAPH FOR OPTIMUM PROCUREMENT CYCLES

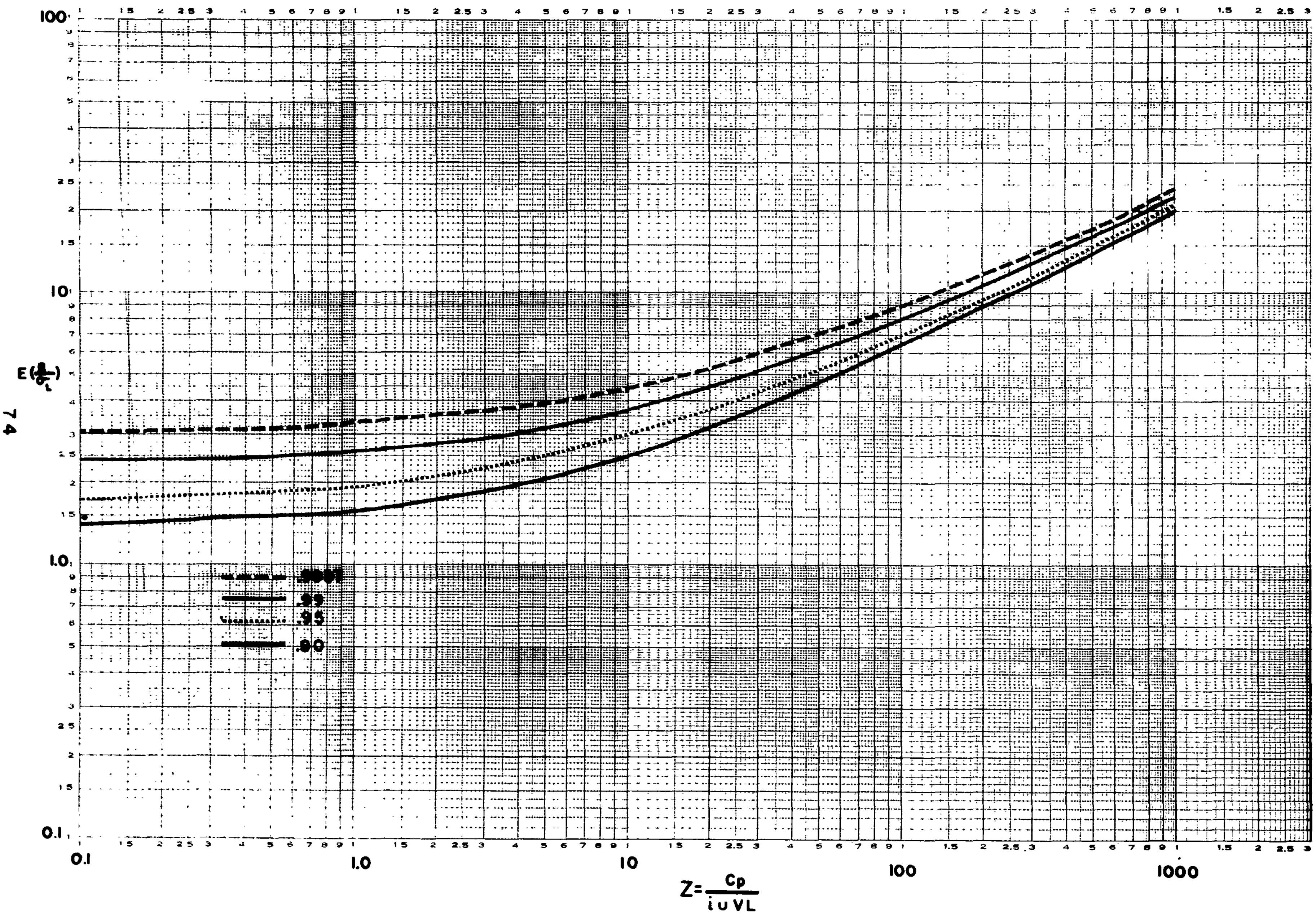


EXHIBIT 24

OPTIMUM PROCUREMENT CYCLE NOMOGRAPH

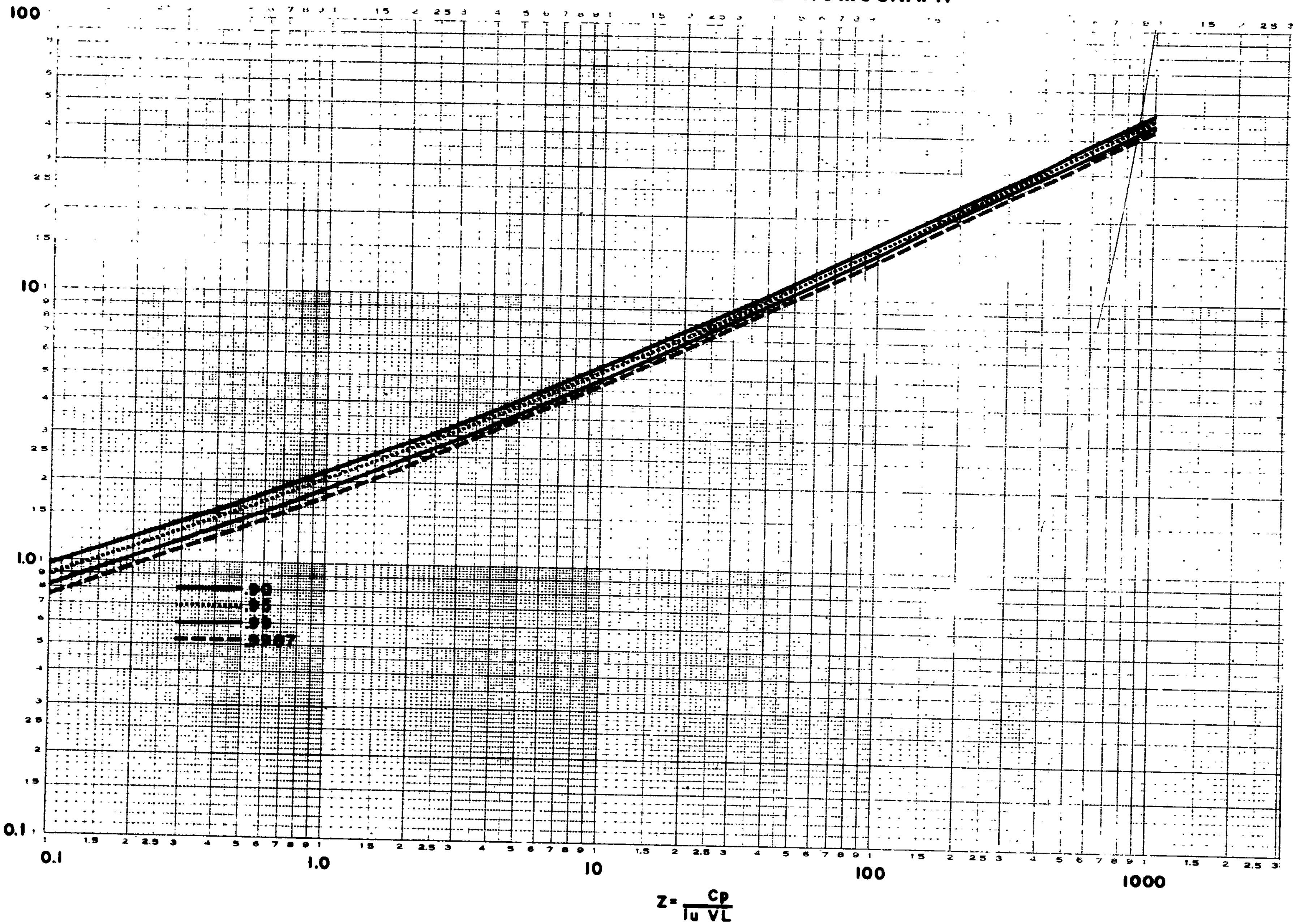
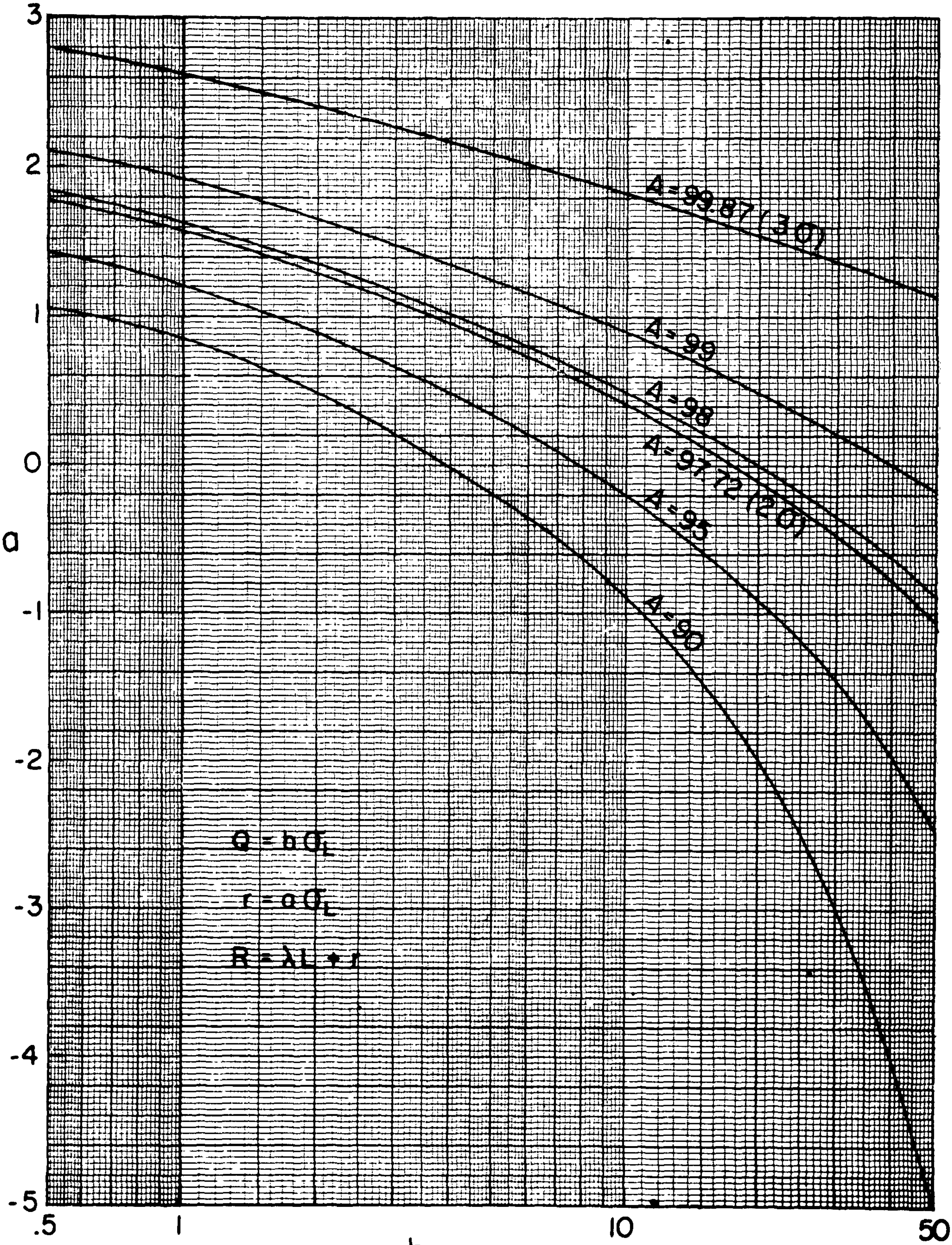


EXHIBIT 25

r-NOMOGRAPH FOR DETERMINATION OF REORDER LEVEL



APPENDIX II

MATHEMATICAL BASIS OF PROPOSED SYSTEM

The mathematical basis for the model used to characterize the national stock of an individual secondary item consists of four parts:

1. The probability distribution of total quantity demanded in a period of time.

2. The probability distribution of national stock derived using 1.,an (R, Q) replenishment policy, and assuming constant replenishment time.

3. Results under the Gaussian approximation to 1.

4. The solution for R given Q when national availability is specified, and for Q under linear assumptions about costs.

These topics are now presented in the above order.

1. <u>The probability distribution of total quantity demanded in a period of time.</u>

Two assumptions are made:

(a) The placing of a <u>demand</u> by a customer is a random process, having a stationary <u>Poisson</u> distribution. Thus if N_T is the total number of demands placed (on <u>all</u> depots and/or NICP's) during a time period of length T, then

$$(1) \quad a(n;\ T) = \text{Prob}\left[N_T = n\right] = \frac{(\lambda T)^n e^{-\lambda T}}{n!}, \quad n = 0,\ 1,\ \ldots$$

It will be noted that λ is the mean number of demands placed per unit time, and λT the mean number in time T. Stationarity involves assuming that the value λ does not change during the time period T.

(b) Each demand consists of an <u>order</u> for a quantity S. The value of S is chosen at random at each demand from a single probability distribution termed the <u>order-size distribution</u>. This distribution is assumed stationary throughout the period of time T. We use the symbolism

$$C(s) = \text{Prob}\left[S \leq s\right]$$

$$(2) \quad S^{(n)} = S_1 + S_2 + \ldots + S_n \text{ (independent observations)}$$

$$\text{and } C_n(s) = \text{Prob}\left[S^{(n)} \leq s\right].$$

While S will often have integer-values, it need not for the consequences we shall draw.

If now we let Y_T denote the total quantity demanded in a period of Time T, and let $F(y;\, T) = \text{Prob}\left[Y_T \leq y\right]$, then it follows that

$$(3) \qquad \begin{aligned} F(0;\, T) &= a(0;\, T) \\ F(y;\, T) &= a(0;\, T) + \sum_{n=1}^{\infty} a(n;\, T)\, C_n(y) \end{aligned}$$

Y_T is thus called a <u>compound-Poisson</u> random variable. We will also refer to it as a <u>stuttering Poisson</u> random variable.

<u>Moments</u>. The moment-generating-function $f(\Theta;\, T)$ of Y_T is easily verified to have the value

$$(4) \qquad \begin{aligned} f(\Theta;\, T) &=_{\text{dfn}} \int_0^{\infty} e^{\Theta y}\, dF(y;\, T) \\ &= e^{-\lambda T\left[1-c(\Theta)\right]} \end{aligned}$$

where $c(\Theta)$ is the moment-generating function of S, i.e., $c(\Theta) =_{\text{dfn}} \int_0^{\infty} e^{\Theta s}\, dC(s)$.

The following consequences are easily verified(*).

$$(5) \qquad \begin{aligned} E(Y_T) &= \text{the mean total quantity demanded in } T = \lambda T E(S) \\ &= (\text{mean number of demands}) \times (\text{mean order size}) \\ \sigma^2(Y_T) &= E\left(\left[Y_T - E(Y_T)\right]^2\right) = \text{variance of } Y_T \\ &= \lambda T\, E(S^2) \end{aligned}$$

Thus an especially interesting result is that the <u>variance to mean</u> ratio, $V(Y_T)$, has the value

$$(6) \qquad V(Y_T) = \frac{\sigma^2(Y_T)}{E(Y_T)} = \frac{E(S^2)}{E(S)}\text{, independently of T.}$$

When S has the geometric distribution,

$$V(Y_T) = 2\, E(s) - 1.$$

(*) E( ) denotes the expected value of the quantity in parentheses.

Further, to study the dispersion of Y_T around its mean, let $\Delta Y_T =$ $\left[Y_T - E(T_T)\right] / \sigma(Y_T)$. Then of course $E(\Delta Y_T^{\,2}) = 1$. We also find that

$$E(\Delta Y_T^{\,3}) = \frac{E(S^3)}{\sqrt{\lambda T}\ \sqrt{E^3\ S^2}}$$

and
$$E(\Delta Y_T^{\,4}) = 3 + \frac{E(S^4)}{\lambda T\ E^2(S^2)}$$

Gaussian Approximation for Large λT. The moment-generating function $g(\theta; T)$ of ΔY_T is

$$g(\theta;\ T) = e^{-\lambda T\left[\frac{E(S)}{\sigma(Y_T)}\ \theta + 1 - c\left(\frac{\theta}{\sigma(Y_T)}\right)\right]}$$

Taking logarithms and utilizing the series expansion of $c(\theta)$:

$$\log g(\theta;\ T) = \lambda T \sum_{n=2}^{\infty} \left[\frac{\theta}{\sigma(Y_T)}\right]^n \frac{1}{n!}\ E(S^n)$$

Thus as $\lambda T \rightarrow \infty$, Y_T has a normal approximation since all terms but the first go to 0. This asymptotic form is the one employed in 3. below.

Demand from various sources and at various locations. In practice one may confront demand from various customer groups; further demand occurs normally at several geographic points. The compound Poisson distribution is especially favorable in such cases because of the fact that if demand from each source is compound Poisson, and the sources generate statistically independent demand, then total quantity demanded is again compound Poisson.

Mathematically this is proved by noting that

$$e^{-\lambda_1 T\left[1 - c_1(\theta)\right]}\ e^{-\lambda_2 T\left[1-c_2(\theta)\right]} = e^{-(\lambda_1 + \lambda_2)T\left[1- \frac{\lambda_1 c_1(\theta)+\lambda_2 c_2(\theta)}{\lambda_1 + \lambda_2}\right]}$$

That is, the sum of two independent compound Poisson random variables is a compound Poisson variable. The customer arrival rates are seen to add (as they should); the net order size is in probability distribution the weighted average (or probability combination) of the two individual distributions of order size, the weights being proportional to the relative frequency of customer demands from the two sources.

Since each of several locations may be thought of as a source, the above argument also applies to demand summed over several locations.

Demand in non-overlapping time periods. If T consists of $T_1 + T_2$, then clearly

$$e^{-\lambda T\left[1-c(\theta)\right]} = e^{-\lambda T_1\left[1-c(\theta)\right]} \; e^{-\lambda T_2\left[1-c(\theta)\right]}$$

Hence Y_{T_1} and Y_{T_2} are statistically independent.

2. The probability distribution of national stock of a secondary item.

The national stock of a given secondary item is assumed to be managed as follows:

(a.) As demands are received they are immediately filled from stock on hand anywhere.

(b.) When such stock is insufficient to fill a demand, the unfilled portion is back-ordered and filled as soon as a replenishment arrives (from manufacture or procurement).

(c.) When the sum of stock on hand + stock on order - quantity back-ordered passes downward across a level R, the reorder level, replenishment action is initiated to add a quantity Q of the item to stock. This quantity Q is called the replenishment quantity.

(d.) The interval of time L from the initiation of a given replenishment action until the corresponding quantity Q arrives into stock is called the replenishment time. This time is assumed to be a fixed constant quantity.

We now make the following definitions:

$M = R + Q$ = maximum stock on hand at any time

= maximum stock on hand and on order at any time

(7) I_t = stock on hand (inventory) at time t

O_t = stock on order at time t

B_t = quantity back-ordered at time t

$A_t = I_t + O_t - B_t$ = net stock assets at time t

$N_t = I_t - B_t$ = net stock at time t.

We note that N_t can be positive or negative, but is always equal either to I_t or $-B_t$.

Because of the fact that all stock on order at time t is received into stock on hand before time t + L, the following fundamental equation holds:

$$(8) \quad N_t = A_{t-L} - Y_L$$

where Y_L is the quantity demanded in the replenishment time L. We note too that A_t and Y_L are statistically independent.

Moreover, under the ordering rules, A_t satisfies the following equation:

$$(9) \quad M - A_t = (M - A_{t-L} + Y_L) \text{ modulo } Q$$

Steady state solution. If the stochastic process of which inventory consists is now studied in the limit as $t \to \infty$, some simple results follow. Accordingly let us drop the subscript t attached to the above variables I_t, O_t, B_t, A_t, N_t, and simple denote their asymptotic values (as $t \to \infty$) by I, O, B, A and N respectively.

The first result follows from a well-known theorem of probability, easily proved here because of the independence of Y_T in every time period T. Namely:

(10) M -A, and therefore A, has a rectangular distribution in the interval R to M. That is, prob $\left[R \leq A \leq R + x\right] = x/Q$.

As a result, equation (8) can be easily solved in the limit as $t \to \infty$ to yield the asymptotic distribution of N. If we let P(x) be the probability that $N \leq x$, we simply get that

$$(11) \quad P(x) = \frac{1}{Q} \int_{M-x}^{\infty} \left[F(y;\ L) - F(y - Q;\ L)\right] dy$$

recalling that F(y: L) denotes the probability that in the replenishment time L the total quantity demanded in the lead time $\leq x$. (Note that if $y < 0$, $F(y;\ L) = 0$).

National availability. Defining national availability as the probability that $N \geq 0$, we have

$$\text{(12) National availability} = 1 - P(0)$$
$$= 1 - \frac{1}{Q} \int_{M}^{\infty} \left[F(y;\ L) - F(y - Q;\ L) \right] dy$$

Average stock on hand. From equation (8), taking expected values on both sides, it follows that

$$\text{Average inventory on hand} = E(I)$$

(13a)
$$= E(A) - E(Y_L) + E(B)$$

= average assets - average demand in L + average backorders

Since average assets equals R + Q/2, (13a) can be written

(13b) $$E(I) = R + \frac{Q}{2} - E(Y_L) + E(B)$$

Expected back-orders. Using (11), the formula for E(B) is

(14)
$$E(B) = \frac{1}{Q} \int_{-\infty}^{0} (-x) \left[F(M-x;\ L) - F(M - x - Q;\ L) \right] dx$$
$$= \frac{1}{Q} \int_{M}^{\infty} (x-M) \left[F(x;\ L) - F(x - Q;\ L) \right] dx$$

3. Results when Y_L has a normal distribution

Let us now consider the case in which Y_L has a normal distribution. This means that if we define $\Delta Y_L = (Y_L - E(Y_L))/\sigma(Y_L)$, then

$$\text{prob} \left[\Delta Y_L \leq x \right] = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-1/2\, t^2}\, dt.$$

We will denote by g(x) the quantity $e^{-x^2/2}/\sqrt{2\pi}$, and by G(x) the quantity $\int_{x}^{\infty} g(y)\, dy$.

Further, we now define a and b by the relations

$$R = E(Y_L) + a\,\sigma_L$$

(15) $$Q = b\,\sigma_L$$

where $\sigma_L = \sigma(Y_L)$

It is then a straightforward matter, although we omit the algebra, to obtain the following results:

(16) $$\text{National availability} = 1 - \frac{(a+b)\,G(a+b) - aG(a) + g(a) - g(a+b)}{b}$$

(17) $$\frac{E(B)}{\sigma_L} = \frac{1}{2b}\left[(1+a^2)G(a) - (1+\left[a+b\right]^2)\,G(a+b) + (a+b)\,g(a+b) - ag(a)\right]$$

and of course we can now write

(18) $$E(I) = \sigma_L\left[a + \frac{b}{2} + \frac{E(B)}{\sigma_L}\right]$$

4. <u>Optimal solution for Q, R for specified P(0) under linear costs.</u>

We now suppose that two costs are present:

(1) a fixed cost C_p incurred every time a replenishment is made

(2) a fixed cost C_I per unit time per unit of average inventory on hand.

We further suppose that 1 - P(0) (national availability) is specified, i.e., that is must equal a certain value α, and that it is now required to find the values of Q and R or therefore just b and a for which total average cost per unit time, denoted C(a, b), is a minimum. Now

$$C(a, b) = C_I \times (\text{average inventory}) + C_p \times \text{frequency of replenishment}$$

$$= C_I\,E(I) + C_p\,\frac{E(Y_L)}{L\,Q}$$

Various procedures are possible. A standard one is to employ a Lagrange multiplier μ, and noting that $E(I)$ and $P(O)$ are functions of a and b, solve the three equations

$$(a)\quad P_o = 1 - \alpha$$

$$(b)\quad C_I \frac{\partial}{\partial a} E(I) = \mu \frac{\partial}{\partial a} P_o$$

$$(c)\quad C_I \frac{\partial}{\partial b} E(I) - \frac{E(Y_L) C_p}{L\, b^2\, \sigma_L} = \mu \frac{\partial}{\partial b} P_o$$

This reduces to solving simultaneously the two equations

$$1 - \alpha = \Big[(a + b)\, G(a + b) - a\, G(a) + g(a) - g(a + b) \Big] / b$$

(21)

$$\text{and } \frac{E(Y_L)\, C_p}{C_I\, L\, \sigma_L^2} = \frac{b}{2}\left[g(a+b) - a\alpha + (a+b)\Big[1 - G(a+b)\Big] - \frac{2b\alpha\Big[1-\alpha-G(a+b)\Big]}{G(a) - G(a+b)} \right]$$

$$- \frac{G(a) - G(a+b)}{2}$$

The b and a nomographs, also an inventory nomograph for $E(I)/\sigma_L$, were obtained by solving these equations. It will be noted that the quantity

$\dfrac{E(Y_L)\, C_p}{C_I\, L\, \sigma_L^2}$ is the entry to a nomograph for b. This quantity bears a close relation to the well-known square root value for economic lot-size.